$$\int \tan\theta\, d\theta = \ln|\sec\theta| + C$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

## Trig to know:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$

$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\frac{d}{d\theta}\tan\theta = \sec^2\theta$

$\frac{d}{d\theta}\sec\theta = \sec\theta\tan\theta$

$\frac{d}{d\theta}\cot\theta = -\csc^2\theta$

$\frac{d}{d\theta}\csc\theta = -\csc\theta\cot\theta$

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$$\int \tan^{-1}(x)\,dx = x\tan^{-1}x - \int \frac{x}{x^2+1}\,dx$$

$u = \tan^{-1}x$, $u' = \frac{1}{x^2+1}$; $v' = 1$, $v = x$

$u = x^2+1$, $du = 2x\,dx$

$\frac{1}{2}\int \frac{2x\,dx}{x^2+1} = \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+1|$

$$x\tan^{-1}x - \frac{1}{2}\ln|x^2+1| + C$$

$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{x^2+1}$

To remember:

$y = \tan^{-1}x$, want $\frac{dy}{dx}$

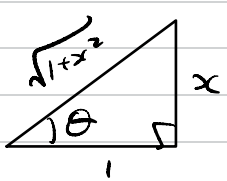
$\rightsquigarrow \tan y = x$

$\rightsquigarrow \sec^2 y \cdot \frac{dy}{dx} = 1$

$\rightsquigarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y$

$= \cos^2\left(\tan^{-1}(x)\right)$ : make triangle

$\theta \rightsquigarrow \tan\theta = \frac{x}{1}$

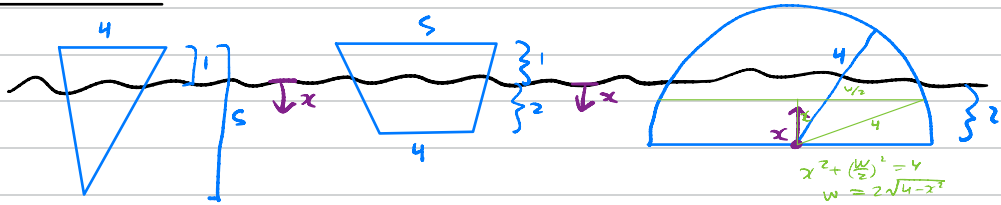



$\cos\theta = \frac{1}{\sqrt{1+x^2}}$, $\cos^2\theta = \frac{1}{1+x^2}$

$= \frac{dy}{dx}$ ✓

# Practice B

4)




(a) (b) (c)

$$F = \int_a^b \rho g\, h(x)\, \overbrace{w(x)\,dx}^{dA}$$

(a) $h(x) = x$, $a = -1$, $b = 4$

$w(x) = 4 + \frac{0-4}{5}(x+1)$

$$\rightsquigarrow \int_{-1}^{+4} \rho g\, x \left(4 - \tfrac{4}{5}(x+1)\right) dx$$

~~$x=-1$~~ 0

(b) $h(x) = x$

$w(x) = 5 + \frac{4-5}{3}(x - -1) = 5 - \frac{1}{3}(x+1)$

$= \frac{14}{3} - \frac{x}{3}$

$$\rightsquigarrow F = \int_{\text{~~0~~}}^{2} \rho g\, x \left(\tfrac{14}{3} - \tfrac{x}{3}\right) dx$$

~~X~~ 0

(c) $w(x) = 2\sqrt{4-x^2}$

$h(x) = 2-x$

$$F = \int_0^2 \rho g (2-x)\, 2\sqrt{4-x^2}\, dx$$