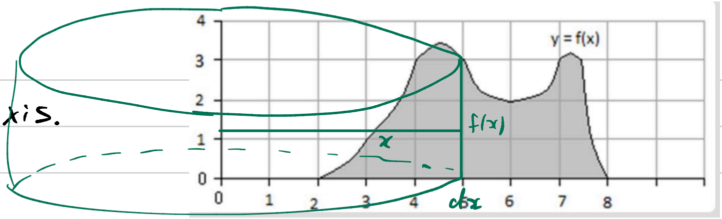
W03 Reg 07

Revolve around y-axis.



$a = 2, \quad b = 8$

$$V = \int_a^b 2\pi R h \, dr$$

$R = x \quad dr = dx \quad h = f(x)$

$$V = \int_2^8 2\pi x f(x) \, dx$$

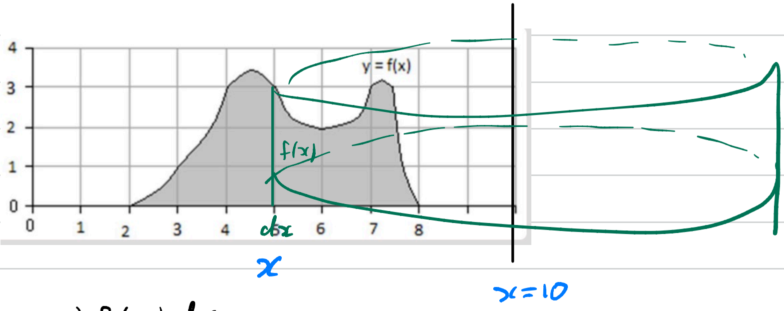
$$V = \int_a^b 2\pi x f(x) \, dx$$

shells around y-axis

Simpson: $S_6 = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + \dots 4y_5 + y_6)$

	$f(x_i)$:	$2\pi x_i f(x_i)$:		$2\pi(10-x_i)f(x_i)$
$x_0 = 2$	0	$2\pi(2)(0)$	$\leftarrow y_0$	$2\pi(8)(0)$
$x_1 = 3$	1	$2\pi(3)(1)$	$\leftarrow y_1$	$2\pi(7)(1)$
$x_2 = 4$	3	$2\pi(4)(3)$	$\leftarrow y_2$	$2\pi(6)(3)$
$x_3 = 5$	3	⋮	⋮	⋮
⋮	⋮			
$x_6 = 8$	0			

Now: what if revolve around $x = 10$?



$$\rightsquigarrow \int_2^8 2\pi(10-x) f(x) \, dx$$

$1+\tan^2=\sec^2$

$$a^2+x^2 \rightsquigarrow x=a\tan\theta$$

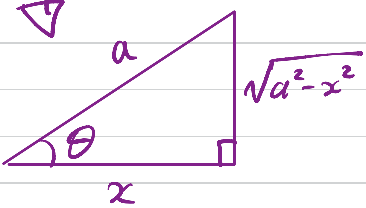
$1-\sin^2=\cos^2$

$$a^2-x^2 \rightsquigarrow x=a\sin\theta$$

$\sec^2-1=\tan^2$

$$x^2-a^2 \rightsquigarrow x=a\sec\theta$$

from triangle



$$\int \frac{1}{x\sqrt{1-x^2}}\,dx \qquad x=1\cdot\sin\theta$$

$$\rightsquigarrow \int \frac{\cos\theta\,d\theta}{\sin\theta\left(\sqrt{\cos^2\theta}\right)}$$

$$=\int \frac{d\theta}{\sin\theta}$$

$$=\int \csc\theta\,d\theta$$

$$=\ln|\csc\theta-\cot\theta|+C$$

$$\int \frac{1}{x\sqrt{x^2-4}}\,dx$$

x^2-2^2

$a=2$

$x=2\sec\theta \rightsquigarrow \frac{x}{2}=\sec\theta \rightsquigarrow \sec^{-1}\left(\frac{x}{2}\right)=\theta$

$dx=2\sec\theta\tan\theta\,d\theta$

$$\rightsquigarrow \int \frac{\cancel{2}\,\cancel{\sec\theta}\,\cancel{\tan}\theta\,d\theta}{\cancel{2}\,\cancel{\sec}\theta\,(2\,\cancel{\tan}\theta)}$$

$$\rightsquigarrow \int \frac{1}{2}\,d\theta=\frac{\theta}{2}+C$$

$$=\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right)+C$$

$$\sqrt{4\sec^2\theta-4}$$

$$=2\sqrt{\sec^2\theta-1}=2\tan\theta$$

$$\int \tan^2 x \sec^2 x \, dx$$

$du = \sec^2 x \, dx$

~~$du = \tan x \sec x \, dx$~~

$u = \tan x$

$$\Rightarrow \int u^2 \, du$$

W02 Reg 07

$$\int_0^{\pi/2} \frac{\cos x \, dx}{\sqrt{1 + \sin^2 x}}$$

$\sin x = u$

$\cos x \, dx = du$

$$\Rightarrow \int_0^1 \frac{du}{\sqrt{1+u^2}}$$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$$\Rightarrow \int_0^{\pi/4} \frac{\sec^2\theta \, d\theta}{\sec\theta}$$

$$\Rightarrow \int_0^{\pi/4} \sec\theta \, d\theta = \dots$$

$\sqrt{1 + \tan^2}$

$= \sqrt{\sec^2}$

$= \sec$

$\int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + C$

$\int \csc\theta \, d\theta = \ln|\csc\theta - \cot\theta| + C$

$$\frac{m}{(x-2)^3(x^2+x+1)(x^2+3)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{Dx+E}{x^2+x+1} + \frac{Fx+G}{x^2+3} + \frac{Hx+I}{(x^2+3)^2}$$

irred?? ✓

$\sqrt{b^2-4ac}$

$b^2-4ac = 1-4(1)(1) = -3 < 0$

imaginary!

$$\int \ln(9x^2-1)\,dx$$

LIATE

$$\begin{array}{l|l} u = \ln(9x^2-1) & v' = 1 \\ \hline u' = \frac{18x}{9x^2-1} & v = x \end{array}$$

$$\hookrightarrow x\ln(9x^2-1) - 18\int \frac{x^2}{9x^2-1}\,dx = x\ln(9x^2-1) - 2 - 3\ln|3x-1| + 3\ln|3x+1| + C$$

$$\begin{array}{r|l} & \frac{1}{9} + \frac{1/9}{9x^2-1} \\ \hline 9x^2-1 & x^2 + 0x + 0 \\ & x^2 \ldots -\frac{1}{9} \\ \hline & +\frac{1}{9} \end{array}$$

$$\frac{x^2 - \frac{1}{9} + \frac{1}{9}}{9x^2-1} = ✓$$

$$\frac{1}{9}\int 1 + \frac{1}{(3x-1)(3x+1)}\,dx$$

$$\hookrightarrow \frac{A}{3x-1} + \frac{B}{3x+1} \rightsquigarrow 1 = A(3x+1) + B(3x-1)$$

$@x = -\frac{1}{3}$: $1 = 0 + B(-2)$, $B = -\frac{1}{2}$

$@x = +\frac{1}{3}$ $1 = A(2) + 0$, $A = +\frac{1}{2}$

$$\hookrightarrow \frac{1}{9}\int 1 + \frac{1/2}{3x-1} + \frac{-1/2}{3x+1}\,dx \rightsquigarrow \frac{1}{9}\left(1 + \frac{1}{6}\ln|3x-1| - \frac{1}{6}\ln|3x+1|\right)$$