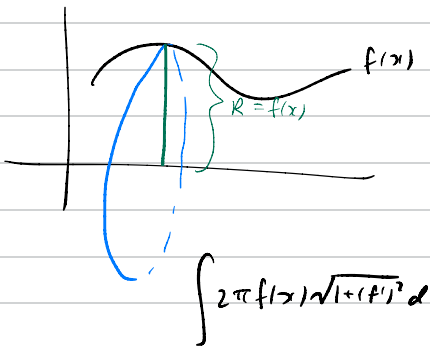
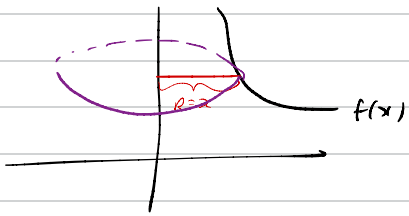
11 am

2/23

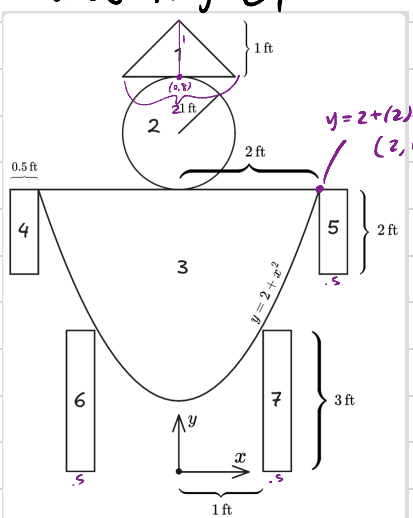


$$\int 2\pi f(x)\sqrt{1+(f')^2}\,dx$$



$$\int 2\pi x\sqrt{1+(f')^2}\,dx$$

W06 Reg 01



$m_1 = \rho \frac{1}{2} 2(1) = 1$

$M_{1y} = 0$

$M_{1x} = ?$ $\quad \bar{x}_1 = 0,\ \bar{y}_1 = 2\frac{5}{3}$

$= 1 \cdot \frac{25}{3} = 2\frac{5}{3}$

$m_4 = m_5 = 1 \qquad m_2 = \pi$

$m_6 = m_7 = 3/2$

$$m_3 = \int_{-2}^{+2} 6-(2+x^2)\,dx = \int_{-2}^{2} 4-x^2\,dx = \frac{32}{3}$$

Skip all $M_{i,y}$ bc $\bar{x} = 0$.

$M_{4x} = m_4 \cdot \bar{y}_4 = (1)(5) = 5$

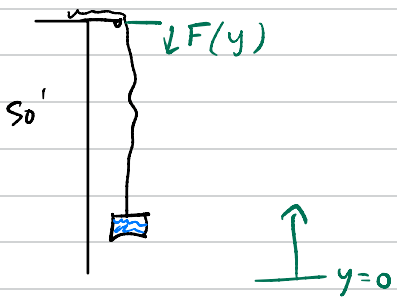
$M_{5x} = 5 \qquad M_{6x} = M_{7x} = m_6 \cdot \bar{y}_6 = \left(\frac{3}{2}\right)\left(\frac{3}{2}\right) = 9/4$

$M_{2x} = (\pi)(7) \qquad M_{3x} = \rho \int_{-2}^{2} \frac{1}{2}\left((6^2)-(2+x^2)^2\right)dx = \frac{704}{15}$

Add all: $M = M_1 + M_2 + \dots + M_7 = \frac{50}{3} + \pi$, $\quad M_x = M_{1x} + \dots + M_{7x} = \frac{2093+210\pi}{30}$

$$\rightsquigarrow (\bar{x}, \bar{y}) = \left(0, \frac{2093+210\pi}{500+30\pi}\right)$$

WOS Reg 04



bucket = 4 lbs
water starts = 30 lbs
Cham = 0.25 lbs/ft
leak = 0.3 lbs/sec
lift = 3 ft/sec

Solution:

$$W = \int_0^{50} F(y)\,dy$$

$$F = F_b + F_w + F_c$$

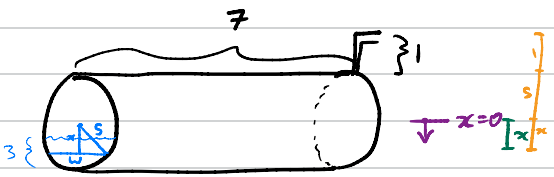
F_b = 4 lbs

$F_c \rightarrow 0.25(50-y)$

$F_w \rightarrow 30 - \frac{0.3 \text{ lb/sec}}{3 \text{ ft/sec}} \cdot y = 30 - 0.1y$

$$\int_0^{50} 4(30-0.1y)(0.25(50-y))\,dy = 1887.5 \text{ ft-lb}$$

WOS Reg OS(6)



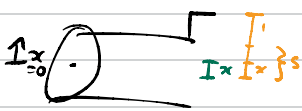
1. Coord (see pic)
2. $h(x) = 6+x$
3. $w(x) = 2\sqrt{25-x^2}$
4. $A(x) = 14\sqrt{25-x^2}$
5. $a = 2,\ b = 5$

$x^2 + (w/2)^2 = 5^2$

$w = 2\sqrt{25-x^2}$

$$W = \int_2^5 \rho g (6+x)(14\sqrt{25-x^2})\,dx$$

Alternate:



$h(x) = 6-x$

$a = -5,\ b = -2$

using "h/3 trick"

12 am

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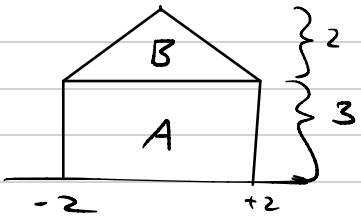
WOG Step 01

assume $\rho = 1$



$m_A = 4 \cdot 3 = 12$

$m_B = \frac{1}{2} \cdot 4 \cdot 2 = 4$

$m = 16$

$M_{A_y} = M_{B_y} = 0$ because symmetry $(\bar{x}_A = \bar{x}_B = \bar{x} = 0)$

$M_{A_x} = ?$ $\quad \bar{y}_A = 3/2$, $\quad M_{A_x} = m_A \cdot \bar{y}_A = 12 \cdot 3/2$

↳ 18

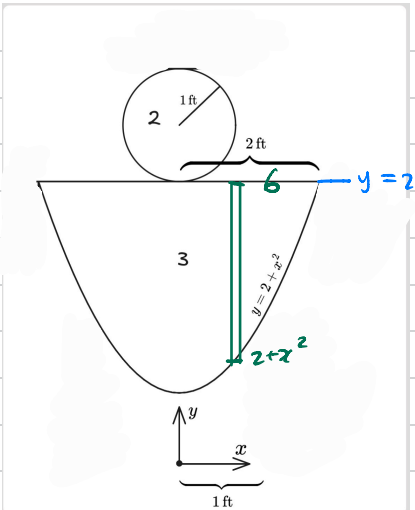
$M_{B_x} = ?$ $\quad \bar{y}_B = 2/3 + 3 = 11/3$, $\quad M_{B_x} = m_B \cdot \bar{y}_B = 4 \cdot 11/3$

↳ $\frac{44}{3}$

Thus: $M_x = M_{A_x} + M_{B_x} = 18 + \frac{44}{3} = \frac{98}{3}$

$$\bar{y} = \frac{M_x}{m} = \frac{98/3}{16} = \frac{49}{24}$$

$$\bar{x} = 0$$



Find CoM of body + head only.

$m_2 = \rho\pi$

$m_3 = \rho\int_{-2}^{2}(6-(2+x^2))\,dx = \rho\frac{32}{3}$

$M_{2_x} = m_2 \cdot \bar{y}_2 = (\rho\pi)(7)$

$M_{3_x} = \rho\int_{-2}^{2}\frac{1}{2}\left(6^2-(2+x^2)^2\right)dx$

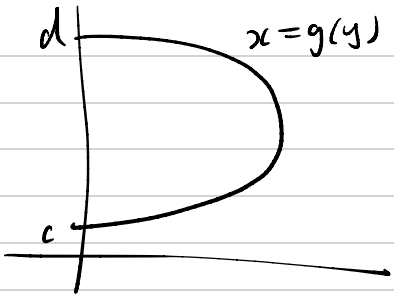
$= \frac{704}{15}\rho$

$m = \rho\left(\pi + \frac{32}{3}\right)$

$M_x = M_{2_x} + M_{3_x} = \rho\left(\frac{32}{3} + \frac{704}{15}\right)$

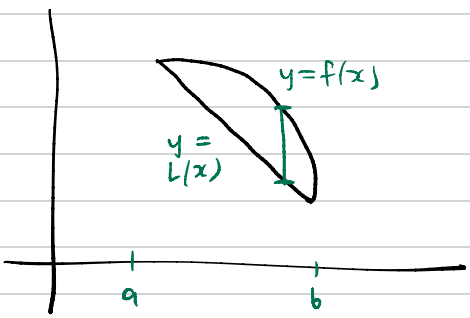
$\bar{x} = 0$

$\bar{y} = \frac{M_x}{m} = \frac{\frac{32}{3}+\frac{704}{15}}{\pi+\frac{32}{3}} = \boxed{\frac{160+704}{15\pi+160}}$



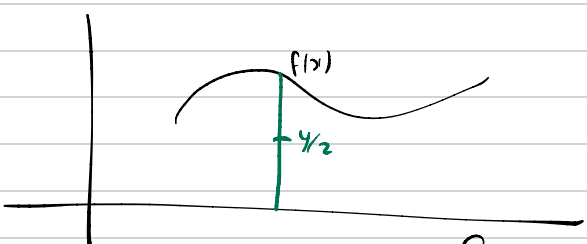
$$M_x = \int_c^d \rho y\,(g(y) - 0)\,dy$$

$$M_y = \rho \int_c^d \frac{1}{2}\left(g(y)^2 - 0^2\right) dy$$



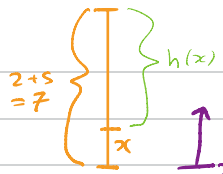
$$M_x = \int_a^b \frac{\rho}{2}\left(f(x)^2 - L(x)^2\right) dx$$

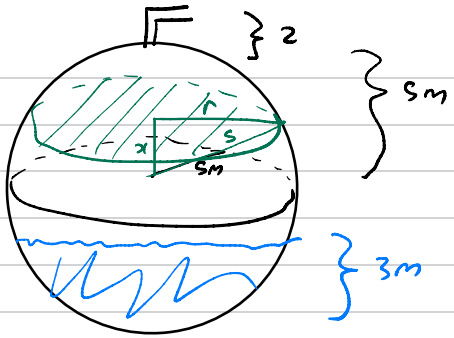
$$M_y = \int_a^b \rho x\,(f(x) - L(x))\,dx$$



$$\rho \int_{\frac{f(x)}{2}} f(x)\,dx = \int \frac{\rho}{2} f(x)^2\,dx$$

WOS Notes





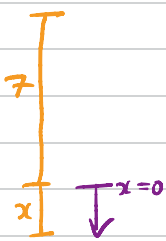
1. Coord.
2. $h(x) = 7 - x$
3. $A(x) = \pi(25 - x^2)$
4. $a = -2,\ b = -5$

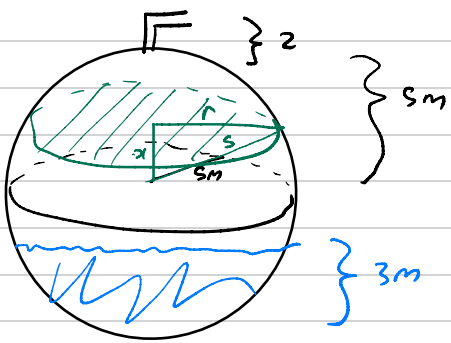
$$W = \int_{-2}^{-5} \rho g (7-x)\pi(25-x^2)\,dx$$

$r^2 + x^2 = 5^2$

$r = \sqrt{25 - x^2}$

$A(x) = \pi r^2 = \pi(25 - x^2)$





$h(x) = 7 + x$

$a = 2,\ b = 5$

$A(x) = \pi(25 - x^2)$

$$W = \int_{2}^{5} \rho g (7+x)\pi(25-x^2)\,dx$$
