W11 Reg 02 (b)

$f(x) = \frac{1}{1+x^4}$ want $\int f\,dx = \sum a_n x^n$

Solution:

Recall $\frac{1}{1-u} = \sum_{n=0}^{\infty} u^n$

$$\frac{1}{1+x^4} \rightsquigarrow \frac{1}{1-(-x^4)} = \sum_{n=0}^{\infty} (-x^4)^n = \sum_{n=0}^{\infty} (-1)^n x^{4n}$$

$= f(x)$

$$\int \frac{1}{1+x^4}\,dx = \int \sum_{n=0}^{\infty} (-1)^n x^{4n}\,dx$$

$$\rightsquigarrow \sum_{n=0}^{\infty} (-1)^n \int x^{4n}\,dx$$

$$\rightsquigarrow \boxed{C + \sum_{n=0}^{\infty} (-1)^n \frac{1}{4n+1} x^{4n+1}}$$

W11 Reg 05

(a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!}$

$$\rightsquigarrow \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} \left(\frac{\pi}{4}\right)^{2n+1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1} \quad @ \; x = \frac{\pi}{4}$$

$$= \sin(x) \quad @ \quad x = \pi/4$$

$$= \sin(\pi/4) = \frac{\sqrt{2}}{2}$$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n}}{n!}$ see: $\frac{1}{n!}$ think: $e^x = \sum \frac{x^n}{n!}$

$$\rightsquigarrow \sum_{n=0}^{\infty} \frac{4^n}{n!} = e^x \Big|_{x=4} = e^4$$

(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+2}}{3^{2n+1}(2n)!}$ see: $\frac{1}{(2n)!}$ think: $\cos(x) = \sum (-1)^n \frac{x^{2n}}{(2n)!}$

So factor: $\frac{\pi^2}{3} \left(\frac{\pi}{3}\right)^{2n}$

$$\rightsquigarrow \frac{\pi^2}{3} \sum_{n=0}^{\infty} (-1)^n \frac{(\pi/3)^{2n}}{(2n)!}$$

W11 Req 6 (d)

$$\sum (-1)^n \frac{\pi^{2n}}{9^n (2n)!} \quad \text{cos} \qquad \cos x = \sum (-1)^n \frac{x^{2n}}{(2n)!}$$

write $\left(\frac{\pi}{3}\right)^{2n}$

ANS $\cos(\pi/3) = \frac{1}{2}$

W11 Req 10

$\cos(0.02)$ w/ error $< 10^{-6}$.

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\cos(0.02) = \sum (-1)^n \frac{(0.02)^{2n}}{(2n)!}$$

$$\rightsquigarrow 1 - \frac{0.02^2}{2!} + \frac{0.02^4}{4!} - \frac{0.02^6}{6!}$$

2×10^{-4} $\qquad$ $6.7 \times 10^{-9} < 10^{-6}$

"Next Term Bound"

$$\rightsquigarrow 1 - \frac{0.02^2}{2!} = 1 - 2 \times 10^{-4} = \boxed{0.9998}$$

W09 Reg 03

(b) $\sum_{n=1}^{\infty} \frac{n^3}{n^5+4n+1} = \sum_{n=1}^{\infty} a_n$

set $b_n = \frac{1}{n^2} = \frac{n^3}{n^5}$

Observe $a_n < b_n$ because denom. of a_n bigger

$\sum b_n$ converges $(p=2)$
so by DCT, $\sum a_n$ converges.

Alternate notation:

$$\frac{n^3}{n^5+4n+1} < \frac{n^3}{n^5} = \frac{1}{n^2} \quad \text{because } n^5+4n+1 > n^5$$

$$\sum \frac{1}{n^2} \text{ conv.} \underset{(DCT)}{\Longrightarrow} \sum \frac{n^3}{n^5+4n+1} \text{ conv.}$$

(c) $\sum_{n=2}^{\infty} \frac{n^2}{n^4-1} = \sum_{n=2}^{\infty} a_n$

(both pos.) $\dfrac{a_n = \frac{n^2}{n^4-1}}{b_n = \frac{n^2}{n^4}} = \frac{n^2}{n^4-1} \cdot \frac{n^4}{n^2} = \frac{n^4}{n^4-1} \xrightarrow{n\to\infty} 1 = \text{"}L\text{"}$

$0 < L < \infty$ so LCT applies

Know $\sum \frac{n^2}{n^4} = \sum \frac{1}{n^2}$ conv. $(p=2)$

By LCT, $\sum a_n$ conv. too.

- CC = Conditionally Convergent
- D = Divergent

a_n	$\lim_{n\to\infty} a_n$	$\{a_n\}$ **C or D**	$\lim_{n\to\infty} (-1)^n a_n$	$\{(-1)^n a_n\}$ **C or D**	$\sum a_n$ **AC, CC, or D**	$\sum (-1)^n$ **AC, CC or D**
$\frac{2n}{n+2}$						
$\frac{3^n}{n^2}$						
$\left(\frac{2n+8}{n^2}\right)^n$						
$\frac{n}{n^2+3}$						
$\frac{n!}{100^n}$						
$\frac{1}{1+\ln n}$						
$5\left(\frac{3}{4}\right)^n$						
$\frac{10^n}{n!}$						