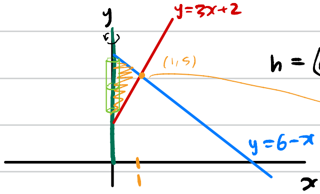
1/12/26

Example region between: $y = 3x+2$, $y = 6-x$, $x = 0$ rotated around y-axis

Solution:



$h = (6-x)-(3x+2)$
$= -4x+4$

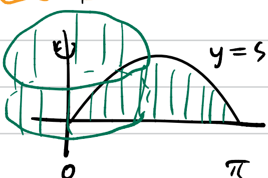
Solve for intersection:
$3x+2 = 6-x$
$4x = 4$
$x = 1 \rightsquigarrow y = 5$
$(1,5)$

1) Shells: $V = \int_a^b 2\pi R h \, dr$

2) Interpret: $r = x$ (because rotate around y-axis)
$R(r) = r = x$, $h = -4x+4$ $dr = dx$
$a = 0$, $b = 1$

$$V = \int_0^1 2\pi(x)(-4x+4)\,dx$$

Example first hump of $y = \sin x$ about y-axis



$$V = \int_0^\pi 2\pi(x)(\sin x)\,dx = 2\pi\int_0^\pi x\sin x\,dx$$

Hint: $\frac{d}{dx}(x\cos x) = 1\cdot\cos x + x(-\sin x)$

$\rightsquigarrow x\sin x = \cos x - \frac{d}{dx}x\cos x$

$\rightsquigarrow \int x\sin x\,dx = \int \cos x\,dx - \int \frac{d}{dx}x\cos x\,dx$

$= \sin x + C_1 - x\cos x + C_2$

$= \sin x - x\cos x + C$

$\rightsquigarrow 2\pi\int_0^\pi x\sin x\,dx = 2\pi\left((\sin\pi - \pi\cos\pi) - (\sin 0 - 0\cos 0)\right)$

$= 2\pi^2$

1 $\int x e^x\,dx$ 2 $\int x\cos x$ 3 $\int \sin^{-1}(x)\,dx$ 4 $\int x\sqrt{x+1}\,dx$

5 $\int \ln x\,dx$ 6 $\int x^4 e^x\,dx$

All Products

3: $\int \sin^{-1}x\,dx = \int 1\cdot\sin^{-1}x\,dx$

5: $\int \ln x\,dx = \int 1\cdot\ln x\,dx$

Method:

$$\int uv'\,dx = uv - \int u'v\,dx$$

Alternate: $\int u\,dv = uv - \int v\,du$

E.g. $\int x e^x\,dx$

1. Choose u, v': $u = x$, $v' = e^x$

2. Make chart:

$\frac{d}{dx}$

$u = x$	$v' = e^x$
$u' = 1$	$v = e^x$

$\int v' = \int e^x\,dx = e^x$

3. Plug in formula:

$$\int uv'\,dx = \int x e^x\,dx = x e^x - \int 1\cdot e^x\,dx$$

$$= x e^x - \int e^x\,dx = \boxed{x e^x - e^x + C}$$

$$\int x\cos x\,dx$$

$$\begin{array}{c|c} u = x & v' = \cos x \\ \hline u' = 1 & v = \sin x \end{array}$$

$$= x\sin x - \int 1\cdot\sin x\,dx$$

$$= \boxed{x\sin x + \cos x + C}$$

$$\int x^4 e^x\,dx = x^4 e^x - \int 4x^3 e^x\,dx$$

$$= x^4 e^x - \left(4x^3 e^x - \int 12x^2 e^x\,dx\right)$$

$$12x^2 e^x - \int 24x e^x\,dx$$

$$24x e^x - \int 24 e^x\,dx$$

$$24 e^x + C$$

$$\boxed{= x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24x e^x + 24 e^x - C}$$

$$\int 1 \cdot \ln x \, dx$$

$$1 \xrightarrow{d} 0 \qquad 1 \xrightarrow{\int} x$$

$$\ln x \xrightarrow{d} 1/x \qquad \ln x \xrightarrow{\int} ??$$ wait....

$u = \ln x$	$v' = 1$
$u' = 1/x$	$v = x$

$$\int \ln x \, dx = x \ln x - \int \frac{1}{x} x \, dx$$

$$\boxed{= x \ln x - x + C}$$

$$\int x \ln x \, dx$$

$u = \ln x$	$v' = x$
$u' = 1/x$	$v = x^2/2$

$$= \frac{x^2}{2} \ln x - \int \underbrace{\frac{1}{x} x^2/2}_{= \frac{x}{2}} \, dx$$

$$\boxed{= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C}$$

← more "u" | more "v" →

L I A T E

Log

Inverse trig

Algebraic

Trigonometric

Exponential

$$\int \sin^{-1} x \, dx$$

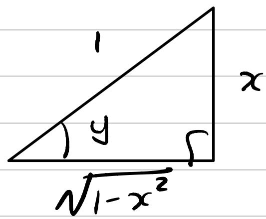
$u = \sin^{-1} x$	$v' = 1$
$\frac{1}{\sqrt{1-x^2}}$	$v = x$

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^{-1}(x) = y$$

$$x = \sin(y)$$

$$1 = x' = \cos(y) \frac{dy}{dx}$$



$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\sqrt{1-x^2}}$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

Easier!

$u = 1 - x^2$

$du = -2x \, dx$

$$\int \frac{-\frac{1}{2} du}{\sqrt{u}} = \ldots$$