

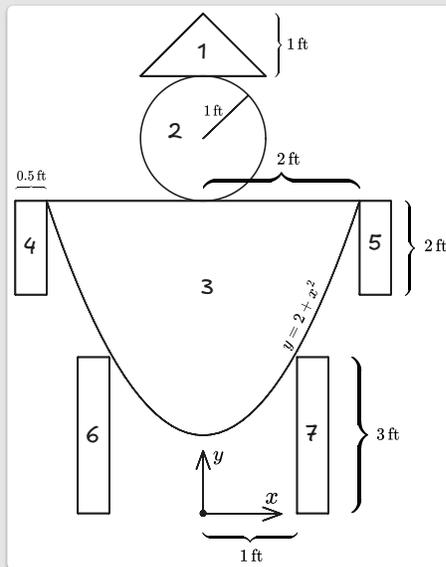
W06 Regular

Due date: Sunday 2/22, 11:59pm

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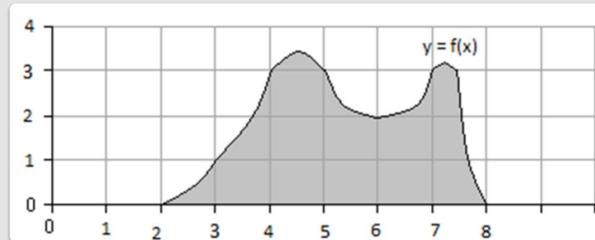
FlatCoMMan

Find the center of mass of FlatCoMMan. Assume a constant mass density ρ . Use additivity of moments.



✍ CoM from Simpson's

Use Simpson's rule (with 6 subintervals) to estimate the centroid of this region:



You will need to estimate M_x and M_y and M with three separate integrals. You can use a calculator for your arithmetic.

✍ Proper vs. improper

For each integral below, determine whether it is proper or improper, and if improper, explain why.

(a) $\int_0^2 \frac{dx}{x^{1/3}}$ (b) $\int_0^1 e^{-x} dx$ (c) $\int_0^{\pi/2} \sec x dx$

(d) $\int_0^\infty \sin x dx$ (e) $\int_1^\infty \ln x dx$ (f) $\int_0^3 \ln x dx$

✍ Gabriel's Horn - Volume and surface of revolution

The curve $y = \frac{1}{x}$ for $x > 1$ is rotated about the x -axis. The resulting shape is *Gabriel's Horn*.

- (a) Find the volume enclosed by the horn by evaluating a convergent improper integral.
- (b) Show that the surface area of the horn is infinite by applying comparison to a p -integral which is divergent.

✍ Comparison test

Use the comparison test to determine whether the integral converges:

$$\int_1^{\infty} \frac{dx}{x^4 + \sqrt{x}}$$

✍ Computing improper integrals

For each integral below, give the limit interpretation of improper integral and then compute the limit. Based on that result, state whether the integral converges. If it converges, what is its value?

(a) $\int_0^1 \ln x \, dx$ (b) $\int_{-\infty}^{+\infty} x e^{-x^2} \, dx$ (c) $\int_1^2 \frac{1}{(x-1)^2} \, dx$