Sequences

Def: $a_0, \quad a_1, \quad a_2, \quad a_3, \quad \ldots, \quad a_n, \quad \ldots$

(a_0: first term; a_n: "general term")

Def: Geometric Sequence:

$$\frac{a_{n+1}}{a_n} = r = \text{constant ratio of consecutive terms (for all } n\text{)}$$

Therefore: $a_0 \xrightarrow{\times r} a_1 \xrightarrow{\times r} a_2 \xrightarrow{\times r} a_3 \xrightarrow{\times r} a_4 \quad \ldots$

$a_1 = a_0 r$

$a_2 = a_1 r = a_0 r^2$

$a_3 = a_2 r = a_1 r^2 = a_0 r^3$

i.e. $a_n = a_0 r^n$ "general term formula"

Examples:

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, 1 \cdot \left(\frac{1}{2}\right)^n, \ldots$

$a_0 = 1, \quad r = 1/2$

$\pi, \pi^2, \pi^3, \ldots, 1 \cdot \pi^n, \ldots; \quad r = \pi$

$5, -\frac{1}{8}, \frac{1}{320}, \ldots$

$r = \frac{-1/8}{5} = -\frac{1}{40}$

$\rightsquigarrow \left(-\frac{1}{8}\right)\left(-\frac{1}{40}\right) = \frac{1}{320}$

$\frac{3}{2}, -\frac{1}{2}, \frac{1}{6}, -\frac{1}{18}, \ldots$

$r = \frac{-1/2}{3/2} = -\frac{1}{3}$

$\rightsquigarrow \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right) = \frac{1}{6}$

$\rightsquigarrow \left(\frac{1}{6}\right)\left(-\frac{1}{3}\right) = -\frac{1}{18}$

$(a_n = a_0 r^n)$

<u>Examples:</u> (a) $a_n = \left(-\frac{1}{2}\right)^n$ (b) $b_n = -3\left(\frac{2^{n+1}}{5^n}\right)$

(c) $c_n = e^{5+7n}$. Find first and r.

Write general term in <u>standard form</u>.

<u>Solution:</u> (a) $a_0 = 1$, $r = -\frac{1}{2}$, $a_n = 1 \cdot \left(-\frac{1}{2}\right)^n$

(b) $b_0 = -3\left(\frac{2^{0+1}}{5^0}\right) = -3\left(\frac{2}{1}\right) = -6$

$$\frac{b_{n+1}}{b_n} = \frac{-3\left(\frac{2^{n+1+1}}{5^{n+1}}\right)}{-3\left(\frac{2^{n+1}}{5^n}\right)} = \frac{2^{\cancel{n+2}\,1}}{5^{\cancel{n+1}}} \cdot \frac{5^{\cancel{n}}}{2^{\cancel{n+1}}} = \frac{2}{5} = r$$

$$b_n = -6\left(\frac{2}{5}\right)^n$$

(c) $c_0 = e^5$, $r = e^7$

$$e^{5+7n} = e^5 e^{7n} = \underbrace{(e^5)}_{c_0}\underbrace{(e^7)}_{r}{}^n$$

$$c_0 = e^{5+7\cdot 0} = e^5$$

$$\frac{c_{n+1}}{c_n} = \frac{e^{5+7(n+1)}}{e^{5+7n}} = \frac{e^{5+7n+7}}{e^{5+7n}} = e^7$$

($e^{5+7n+7} = e^{5+7n} \cdot e^7$)

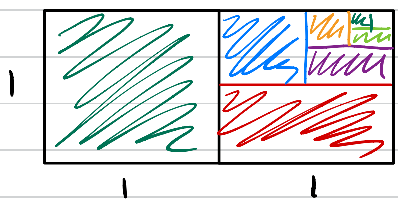
Series

Def: $\sum_{n=0}^{\infty} a_n = a_0 + a_1 + a_2 + a_3 + \dots + a_n + \dots$

Examples: $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \dots = \frac{\pi}{4}$

"Leibniz Series" GOAT

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \left(\frac{1}{2}\right)^n + \dots = 2$



Geometric Series
$a_0 = 1, \quad r = 1/2$

Total Sum of Geometric Series

Write "S" for the total sum:

$$S = a_0 + a_0 r + a_0 r^2 + a_0 r^3 + \dots = \sum_{n=0}^{\infty} a_0 r^n$$

$$-\left(rS = a_0 r + a_0 r^2 + a_0 r^3 + a_0 r^4 + \dots \right)$$

$$S - rS = a_0 \rightsquigarrow S(1-r) = a_0$$

$$\rightsquigarrow \boxed{S = \frac{a_0}{1-r}}$$

$$S = \frac{1}{1-1/2} = \frac{1}{1/2} = \boxed{2}$$