W11 Regular

Due date: Sunday 3/29, 11:59pm

01

Power series of a derivative

Suppose that a function $f(x)$ has power series given by:

$$f(x) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{n+1}$$

The radius of convergence of this series is $R = 1$.

What is the power series of $f'(x)$ and what is its interval of convergence?

02

Modifying and integrating a power series

(a) Modify the power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$ to obtain the power series for $f(x) = \frac{1}{1+x^4}$.

(b) Now integrate this series to find the power series for $\int f(x)\,dx$.

03 ★

Maclaurin series

For each of these functions, find the Maclaurin series and the interval on which the expansion is valid.

(a) $\sin(3x^2)$ (b) x^2e^{5x}

04

Taylor series of $1/x$

Find the Taylor series for the function $f(x) = \frac{1}{x}$, centered at $c = 1$, by differentiating repeatedly to determine the coefficients.

05

Discovering the function from its Maclaurin series

For each of these series, identify the function of which it is the Maclaurin series, and evaluate the function at an appropriate choice of x to find the total sum for the series.

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!}$ (b) $\sum_{n=0}^{\infty} \frac{2^{2n}}{n!}$ (c) $\sum_{n=0}^{\infty}(-1)^n \frac{\pi^{2n+2}}{3^{2n+1}(2n)!}$

06 ★

Summing a Maclaurin series by guessing its function

For each of these series, identify the function of which it is the Maclaurin series:

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{5x^{4n+2}}{(2n+1)!}$ (b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-5x)^{n+1}}{n+1}$

Now find the total sums for these series:

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-5)^n}{n!}$ (d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{9^n(2n)!}$

(Hint: for (c)-(d), do the process in (a)-(b), then evaluate the resulting function somewhere.)

07

Data of a Taylor series

Assume that $f(3) = 1$, $f'(3) = 2$, $f''(3) = 12$, and $f'''(3) = 3$.

Find the first four terms of the Taylor series of $f(x)$ centered at $c = 3$.

Evaluating series

Find the total sums for these series:

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{\sqrt{3}^{2n+1}}{3^{2n+1}(2n+1)}$ (b) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{1}{5^{n+1}(n+1)}$

09 ★

Large derivative at $x = 0$ using pattern of Maclaurin series

Consider the function $f(x) = x^2 \sin\left(5x^3\right)$. Find the value of $f^{(35)}(0)$.

(Hint: find the rule for coefficients of the Maclaurin series of $f(x)$ and then plug in 0.)

Some estimates using series

Without a calculator, estimate $\cos(0.02)$ (angle in radians) with an error below 10^{-6}.

(Use the error bound formula for alternating series.)

11 ★

Some estimates using series

Find an infinite series representation of $\int_0^1 \sin(x^2)\,dx$ and then use your series to estimate this integral to within an error of 10^{-3}.

(Use the error bound formula for alternating series.)