W11 Stepwise

Due date: Thursday 3/26, 11:59pm

01

Modifying geometric power series

Consider the geometric power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

For this problem, you should modify the series for $\frac{1}{1-x}$.

(a) Write $\dfrac{1}{5-x}$ as a power series and determine its interval of convergence.

(b) Write $\dfrac{1}{16+2x^3}$ as a power series and determine its interval of convergence.

02

Finding a power series

Find a power series representation for these functions:

(a) $f(x) = \dfrac{x^2}{x^4 + 81}$ (b) $g(x) = x^2 \ln(1 + x)$

03

Maclaurin series

For each of these functions, find the Maclaurin series, and the interval on which the expansion is valid.

(a) $x \ln(1 - 5x)$ (b) $x^2 \cos(x^3)$

Discovering the function from its Maclaurin series

For each of these series, identify the function of which it is the Maclaurin series.

(a) $\sum_{n=0}^{\infty}(-1)^n 2^n x^n$ (b) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{3n}}{n!}$

Approximating $1/e$

Using the series representation of e^x, show that:

$$\frac{1}{e} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots$$

Now use the alternating series error bound to approximate $\frac{1}{e}$ to an error within 10^{-3}.