

Probability - Interpretations

Interpretive frameworks

Epistemological probability

Symmetry

Classical theorists hold that probability derives from the symmetry of a situation (dice, coins, poker).

“Principle of Equal Probability”

The theory of chances consists in reducing all events of the same kind to a certain number of equally possible cases, that is to say, to cases whose existence we are equally uncertain of, and in determining the number of cases favourable to the event whose probability is sought. The ratio of this number to that of all possible cases is the measure of this probability, which is thus only a fraction whose numerator is the number of favourable cases, and whose denominator is the number of all possible cases.

Laplace 1814 – Philosophical Essay of Probabilities

- Primary example: games of chance (dice, cards, coins...)
- Challenge: whence “equal probabilities” for possible outcomes?

Logical / evidential

Logicians hold that probability relates the syntactic structure of theory language.

Evidentialists hold that probability relates known evidence to a proposed hypothesis.

Given a scientific hypothesis h , we can intelligibly ask: how probable is h on present evidence? We are asking how much the evidence tells for or against the hypothesis. We are not asking what objective physical chance or frequency of truth h has. A proposed law of nature may be quite improbable on present evidence even though its objective chance of truth is 1. That is quite consistent with the obvious point that the evidence bearing on h may include evidence about objective chances or frequencies. Equally, in asking how probable h is on present evidence, we are not asking about anyone’s actual degree of belief in h . Present evidence may tell strongly against h , even though everyone is irrationally certain of h .

Williamson 2000 – Knowledge and Its Limits

- Primary example: weather events, scientific facts
- Challenge: What is meant by “evidence tells for a hypothesis”?
- Challenge: Cf. Subjective probability

Subjective probability

Beliefs

Bayesians hold that probability measures the degrees of belief of persons.

Your degree of belief in E is p iff p units of utility is the price at which you would buy or sell a bet that pays 1 unit of utility if E , 0 if not E .

By degree of probability, we really mean, or ought to mean, degree of belief.

De Morgan 1847 – Formal Logic, or, The Calculus of Inference, Necessary and Probable

Probability “is a measurement of belief qua basis of action” (F. Ramsey).

Dutch Book Theorem

If your beliefs violate the Kolmogorov Axioms of Probability, then someone can devise a bet you would take and necessarily lose money.

- Primary example: scientific hypotheses and facts
- Challenge: ‘Coherence’ (axioms of probability) leaves much open
- Challenge: doesn’t allow being ‘wrong’ about probability (only rationality)
- Challenge: hard to ascertain beliefs or preferences

Physical probability

Frequencies

Frequentists hold that probability derives from patterns in repeated trials.

Probability of *A* is always relative to a ‘reference class’ *B*. It is the proportion of *actual* outcomes that are *A*. (vs. classical: proportion of *possible* outcomes.)

probability is nothing but that proportion [of births of males and females]

Venn 1876 – The Logic of Chance

- Primary example: statistical tests and studies
- Challenge: One-shot events like decay of a single uranium atom
- Challenge: Reference class problem

Propensities

Propensity theorists hold that probability measures a causal power or tendency inhering in objects.

I am, then, to define the meaning of the statement that the probability, that if a die be thrown from a dice box it will turn up a number divisible by three, is one-third. The statement means that the die has a certain ‘would-be’; and to say that the die has a ‘would-be’ is to say that it has a property, quite analogous to any habit that a man might have.

C.S. Peirce 1910 – Notes on the Doctrine of Chances

- Primary example: actions/habits of living organisms; decay of single uranium atom
- Challenge: “Suppose a sick patient has propensity to trigger a medical test. Do positive medical tests have propensity to have come from a sick patient?”
- Challenge: Other problems shared with Frequentists.

Scenario	Natural framework
Rolling a die	Classical
Do gravitational waves exist?	Evidentialist or Bayesian
Decay of single uranium atom	Propensity
Polling voters	Frequency
Winning at slot machine	Classical

Scenario	Natural framework
Significance test	Frequency
MAP criterion	Bayesian
Tornado occurrence	Evidentialist, Bayesian
Chance that Jane goes to movie	Propensity
Relativity Theory is true	Bayesian, Evidentialist
Chloroquine cures COVID	Frequency
Lab Leak Hypothesis	Bayesian, Evidentialist

Case studies

COVID testing

- Background rate: 1% of population has COVID
- Sensitivity: 95% chance positive test on people *with* COVID
- Selectivity: 90% chance negative test on people *without* COVID

C_p = has COVID

C_n = no COVID = C_p^c

T_p = tests positive

T_n = tests negative = T_p^c

Bayesian approach

Priors:

- $P[C_p] = 0.01$
- $P[T_p | C_p] = 0.95$
- $P[T_n | C_n] = 0.90$

Question

Find $P[C_p | T_p]$.

Solution

Bayesian calculation:

$$P[C_p | T_p] = P[T_p | C_p] \cdot \frac{P[C_p]}{P[T_p]}$$

We may view this formula (rewrite) as “update priors in light of evidence”:

$$P[C_p | T_p] = \frac{P[T_p | C_p]}{P[T_p]} P[C_p]$$

Division into case for denominator:

$$\begin{aligned} P[T_p] &= P[T_p | C_p] P[C_p] + P[T_p | C_n] P[C_n] \\ &= 0.95 \times 0.01 + 0.10 \times 0.99 \approx 0.1085 \end{aligned}$$

Thus:

$$P[C_p | T_p] = 0.95 \times \frac{0.01}{0.1085} \approx 0.088$$

Frequentist approach

Type I error rate: 10%

Type II error rate: 5%

Significance level: 10%

Election polling

Say 1,000 people are polled for the 1836 US election.

52% for Harrison

48% for Van Buren

Frequentist approach

Write p for the true probability for Harrison.

▣ Confidence interval for sample mean

Confidence interval relation:

$$P\left[M_n(X) \in [p - \varepsilon, p + \varepsilon]\right] \geq 2\Phi(2\varepsilon\sqrt{n}) - 1$$

Thus:

$$P\left[0.49 \leq M_n(X) \leq 0.55\right] \geq 95\%$$

Typical imposed “significance level” is 5%. Thus, the poll is *significant* if the 95% window excludes 50% for Harrison. Therefore, this poll is *not significant*.

Bayesian approach

We need to know $P[H_0]$, the prior probability of Harrison winning before the poll is conducted.