A boosted AC circuit is supposed to maintain an average voltage of 130 V with a standard deviation of 2.1 V. Nothing else is known about the voltage distribution.

Design a two-tail test incorporating the data of 40 independent measurements to determine if the expected value of the voltage is truly 130 V. Use $\alpha = 0.02$.

Let X_i = one measurement

$M_{40} = \frac{1}{40} \sum X_i$ = avg of 40

$H_0 = E[V] = E[M_{40}] = 130$

Claim: it's not: $|M_{40} - 130| > 0$

RR: $\{ x \mid |x - 130| \geq c \}$ = Two Tail region

want c such that:

$$P[\,|M_{40} - 130| \geq c\,] = 0.02$$

Use Chebyshev!

$E[M_{40}] = 130$, $\sigma^2_{M_{40}} = \frac{(2.1)^2}{40} = 0.110$

$$P[\,|M_{40} - 130| \geq c\,] \leq \frac{\sigma^2_{M_{40}}}{c^2} = \frac{0.110}{c^2} = 0.02$$

$\rightsquigarrow c = 2.348$

$RR = \{ x \mid |x - 130| \geq 2.348 \}$ (subset of $\mathbb{R}$)

$$= \boxed{M_{40} \leq 127.65, \quad M_{40} \geq 132.35}$$

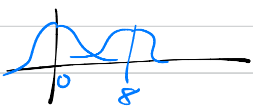
$$f_{X|H_0}(x)\cdot P[H_0] \geq f_{X|H_1}(x)\cdot P[H_1]$$ ← MAP Criterion

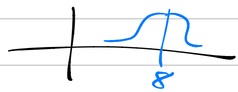
Suppose that a smoke detector sensor is configured to produce 8 V when there is smoke, and 0 V otherwise. But there is background noise with distribution $\mathcal{N}(0, 3^2\text{ V})$.

Design an ML test for the detector electronics to decide whether to activate the alarm.

What are the three error probabilities? (Type I, Type II, Total.)

1. $X|H_0 \sim \mathcal{N}(0, 3^2)$, $X|H_1 \sim \mathcal{N}(8, 3^2)$





2. $f_{X|H_0} = \frac{1}{\sqrt{2\pi 9}}\, e^{-\frac{1}{2}\left(\frac{x-0}{3}\right)^2}$ $f_{X|H_1} = \frac{1}{\sqrt{2\pi 9}}\, e^{-\frac{1}{2}\left(\frac{x-8}{3}\right)^2}$

3. ML: Cancel $P[H_0]$, $P[H_1]$, ML criterion:

$$\frac{1}{\sqrt{2\pi 9}}\, e^{-\frac{1}{2}\left(\frac{x-0}{3}\right)^2} \geq \frac{1}{\sqrt{2\pi 9}}\, e^{-\frac{1}{2}\left(\frac{x-8}{3}\right)^2}$$

$$\cancel{\frac{1}{\sqrt{2\pi 9}}}\, e^{-\frac{1}{2}\left(\frac{x-0}{3}\right)^2} \geq \cancel{\frac{1}{\sqrt{2\pi 9}}}\, e^{-\frac{1}{2}\left(\frac{x-8}{3}\right)^2}$$

ln( ) ln( )

$$\rightsquigarrow -\frac{1}{2}\left(\frac{x-0}{3}\right)^2 \geq -\frac{1}{2}\left(\frac{x-8}{3}\right)^2$$

$$\rightsquigarrow \left(\frac{x-0}{3}\right)^2 \leq \left(\frac{x-8}{3}\right)^2 \rightsquigarrow x^2 \leq (x-8)^2,$$

$x^2 \leq x^2 - 16x + 64,$

$0 \leq -x + 4,$ $x \leq 4$

So $$\boxed{\begin{aligned} A_0 &= \{x \leq 4\} \\ A_1 &= \{x > 4\} \end{aligned}}$$

Type I error:

$$P_{FA} = P[A_1 \mid H_0] = P[x > 4 \mid H_0]$$

$X \mid H_0 \sim \mathcal{N}(0, 3^2)$

$$= 1 - P[X \leq 4 \mid H_0]$$

$$= 1 - P\left[\frac{X-0}{3} \leq \frac{4-0}{3} \,\middle|\, H_0\right]$$

$$= 1 - P[Z \leq 1.33] = \boxed{0.0912}$$

Type II error:

$$P_{Miss} = P[A_0 \mid H_1] = P[X \leq 4 \mid H_1]$$

$X \mid H_1 \sim \mathcal{N}(8, 3^2)$

$$= 1 - P\left[\frac{X-8}{3} \leq \frac{4-8}{3} \,\middle|\, H_1\right]$$

$$= P[Z \leq -1.33] = \boxed{0.0912}$$

(recall: $P[A_1 \mid H_0]\, P[H_0] = P[A_1 H_0]$)

Total error:

$$P_{Err} = \underbrace{P_{FA}}_{P[A_1 \mid H_0]} \cdot P[H_0] + \underbrace{P_{Miss}}_{P[A_0 \mid H_1]} \cdot P[H_1]$$

$$= 0.0912 \cdot (0.5) + 0.0912 \cdot (0.5) = \boxed{0.0912}$$