W03 Homework B

Due date: Tuesday 2/3, 11:59pm

01 ★

Guessing on a test

Your odds of getting any given exam question right are 80%. The exam has 4 questions, and you need to answer 3 correctly to pass.

(a) What is the probability that you pass?

(b) After finishing the exam, you are 100% sure that you got the second question right. Now what are the odds that you pass?

02

Independent trials - At least 45 good paper clips

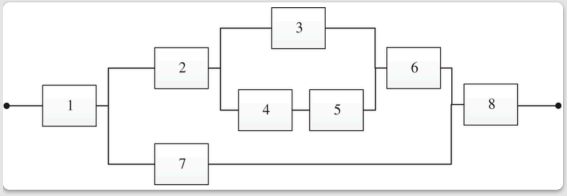
For a paper clip production line, 90% of the paper clips come off good, and 10% come off broken.

You buy a box of 50 paper clips from this line. What is the probability that at least 45 of them are good?

03

Reliability for complex process

Consider a process with the following diagram of components in series and parallel:



Use W_i to denote the event that component i succeeds.

Suppose the success probabilities per component are given by this chart:

1	2	3	4	5	6	7	8
80%	60%	40%	90%	80%	50%	70%	90%

What are the odds of success for the whole process?

04 ★

PMF from CDF for tracking gasoline

I am trying to keep track of how much gasoline (rounded to the nearest integer and denoted by X) my car uses every week. I have managed to find the CDF of X:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.1 & 0 \leq x < 1 \\ 0.2 & 1 \leq x < 2 \\ 0.5 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

Assume gasoline costs \$3 a gallon. Let Y denote the amount of money I spend on gasoline every week. What is the PMF of Y?

05

Gambling with a coin

Two players, A and B, are flipping a fair coin together. If it comes up heads, A pays $1 to B, and if it comes up tails, B pays $1 to A.

They play five rounds. Let X be a random variable recording A's final winnings.

(a) Find the set of possible values of X. (I.e., the set of outcomes with nonzero probability.)

(b) Find the PMF and CDF of X.

06 ★

Digit of a real number

Suppose a real number is chosen randomly in the unit interval $[0, 1]$. Consider the decimal expansion of this number. Let Y be a random variable giving the first digit after the decimal point. Find the possible values, the PMF, and the CDF of Y.

07

Multinomial - Colored marbles in a line

How many ways are there to line up 10 colored marbles (2 red, 3 white, 5 blue), assuming you cannot distinguish marbles of the same color?

08

Multiplication Rule - Fund performance

The odds of the Winning Fund outperforming the market in a random year are 15%. The odds that it outperforms the market in a 1-year period assuming it has done so in the prior year are 30%.

What is the probability of the Winning Fund outperforming the market in 2 consecutive years?

09

Applicant qualifications A

A hiring manager will randomly select two people from a group of 5 applicants. Of the 5 applicants, 2 are more qualified and 3 are less qualified (but the manager does not know this).

If at least one of the less qualified applicants is selected, what is the probability that both applicants selected will be less qualified?

10

Applicant qualifications B

A hiring manager will randomly select two people from a group of 5 applicants. Of the 5 applicants, 2 are more qualified and 3 are less qualified (but the manager does not know this).

Let Event A be selecting 2 more qualified applicants and Event B be selecting 2 less qualified applicants. Determine whether A and B are independent events and justify your answer.