W04 Homework B

01 ★

> ✏ **Prize on the Mall**
>
> A booth on the Mall is running a secret prize game, in which the 5th passerby wearing a hat wins $1,000.
>
> Passersby wear hats independently of each other and with probability 20%.
>
> Let N be a random variable counting how many passersby pass by before a winner is found.
>
> (a) What is the name of the distribution for N? What are the parameters?
>
> (b) What is the probability that the 10th passerby wins the prize?
>
> (c) What is the probability that at least 7 passersby are needed before a winner is found?

✎ Geometric distribution is memoryless

Suppose that $X \sim \text{Geom}(p)$. Derive this equation:

$$P\big[X = n + k \mid X > n\big] \;=\; P[X = k]$$

Interpret the equation. (Inspired by the title.)

☑ Half the babies are female?

At Grace Community Hospital, 4 babies are delivered in one day. At Hope Valley Hospital, 6 babies are delivered in one day.

Consider these two events:

- (i) exactly half the babies born at Grace Community are female
- (ii) exactly half the babies born at Hope Valley are female

Perform a calculation to determine whether Event (i) is more probable, Event (ii) is more probable, or they are equally probable. (Assume the probability of each baby being born male or female is 0.5.)

✏ A very strange car

A very strange car with n components will drive if at least half of its components work. Each component will work with the same probability p, independently of the others.

For what values of p is a car with $n = 3$ more likely to drive than a car with $n = 5$?

(Start by defining a random variable that counts the number of working components.)

✍ Tutoring needs

A course with 6 students offers free one-on-one tutoring to each student for 1 hour the week before the final exam. One tutor, Jim, has been hired to provide this tutoring, but he is available for only 4 hours that week. The instructor of the course will tutor any students that Jim is not able to help. Jim will be paid $20 per hour by the department. The instructor will provide tutoring for free. Let X be the number of students that will need tutoring. The PMF of X is given below.

$$
P_X(x) = \begin{cases} 0.10 & x = 0 \\ 0.20 & x = 1, 2 \\ 0.15 & x = 3, 4, 5 \\ 0.05 & x = 6 \\ 0 & \text{otherwise} \end{cases}
$$

(a) Find the probability the instructor will need to provide tutoring.

(b) Find the expected value of the number of students that will need tutoring.

☑ **Students and buses expect different crowding**

Bus One has 10 students, Bus Two has 20, Bus Three has 30, and Bus Four has 40.

- Let X measure the number of students on a given random student's bus.
- Let Y measure the number of students on a given random driver's bus.

Compute $E[X]$ and $E[Y]$. Are they different? Why or why not?

☑ **Rolling two dice**

Two dice are rolled. Find the probabilities of the following events:

- *A*, the event that the sum is 10
- *B*, the event that the sum is 12
- *C*, the event that the two numbers are equal

✍ Bayes' Theorem - DNA evidence

A crime is committed in a town of 100,000 citizens. After all 100,000 citizens' DNA is analyzed, your friend Jim is found to have a DNA match to evidence at the scene. A forensics expert says that the probability of a random person matching this evidence is 0.01%. How likely is it that Jim is guilty?