# Common Discrete Families

Bernoulli Process : Bernoulli, Binomial, Geometric, Pascal

Poisson Process : Poisson

→ Exponential, Erlang are <u>continuous</u>

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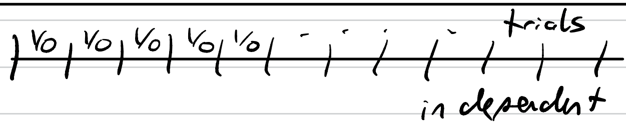
Bernoulli Process:



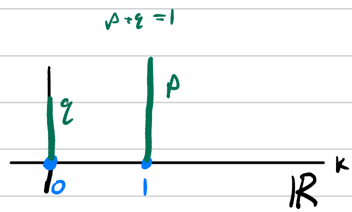

E.g. flip biased coin repeatedly

HTHHTHTTTH . . . . .

"PMF"

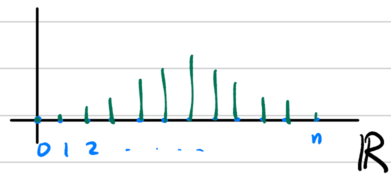
$X_i \sim Ber(p)$, $\qquad P_{X_i}(k) = \begin{cases} p & k=1 \\ q & k=0 \\ 0 & \text{else} \end{cases}$

indicates 1/0 of $i^{th}$ trial




$S \sim Bin(n,p)$, $\qquad P_S(k) = \binom{n}{k} p^k q^{n-k} \qquad k = 0, 1, \ldots, n$

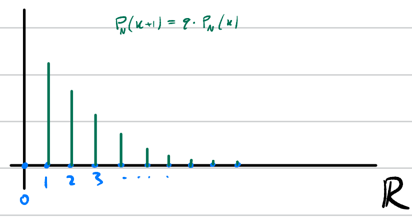
counts # 1 in $n$ trials




Have relationship: $S = X_1 + \ldots + X_n$

$N \sim Geom(p)$, $\qquad P_N(k) = q^{k-1} p \qquad k = 1, 2, 3, \ldots$

counts discrete wait until first 1




Check: $\sum_{k=1}^{\infty} P_N(k) = \sum_{k=1}^{\infty} q^{k-1} p$

$= \text{"}\frac{a}{1-r}\text{"} = \frac{p}{1-q} = \frac{p}{p} = 1 \quad \checkmark$

$\ell = 3, \quad k = 5 \qquad \begin{matrix} 01101\cdots + \\ 11001\cdots + \\ \vdots \end{matrix} \qquad \binom{4}{2} p^1 \cdot p^2 \cdot q^{5-(1+2)}$

$L \sim \text{Pasc}(\ell, p), \qquad P_L(k) = \binom{k-1}{\ell-1} q^{k-\ell} p^{\ell} \qquad k = \ell, \ell+1, \ell+2, \ldots$

counts discrete wait until $\ell^{\text{th}}$ 1

Example: $L \sim \text{Pasc}(3, 1/4)$

then $P_L(8)$ = prob. $3^{\text{rd}}$ win on $8^{\text{th}}$ try

Note: In some texts: $\text{Pasc}(\ell, p) = \text{Negbin}(\ell, p)$

Example: Cubs v. Yankees in World Series

i.e. win 4 of 7 games.

Assume $p = .45$ so $q = .55$. What $P[\text{Cubs win!}] = ?$

Solution: Method (a): Use Binomial:

$X \sim \text{Bin}(7, p) \qquad X = \#$ Cubs' wins

So $\text{ANS} = P_X(4) + P_X(5) + P_X(6) + P_X(7)$

Have: $P_X(k) = \binom{7}{k} p^k q^{7-k}$

$\leadsto \binom{7}{4} p^4 q^3 + \binom{7}{5} p^5 q^2 + \binom{7}{6} p^6 q^1 + \binom{7}{7} p^7 q^0$

$\leadsto \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} p^4 q^3 + \frac{7 \cdot 6}{2} p^5 q^2 + \frac{7}{1} p^6 q^1 + 1 \cdot p^7 q^0$

$\leadsto p^4 (35 q^3 + 21 p^1 q^2 + 7 p^2 q + p^3) \leadsto 0.39$

## Method (b):

$Y \sim \text{Pasc}(4, p)$ $\quad Y = \#$ games till Cubs' $4^{th}$ win.

$$\text{ANS} = P_Y(4) + P_Y(5) + P_Y(6) + P_Y(7)$$

$$P_Y(k) = \binom{k-1}{4-1} p^4 q^{k-4}$$

$$\rightsquigarrow \binom{3}{3} p^4 q^0 + \binom{4}{3} p^4 q^1 + \binom{5}{3} p^4 q^2 + \binom{6}{3} p^4 q^3$$

$$\rightsquigarrow 1 \cdot p^4 q^0 + \frac{4}{1} p^4 q^1 + \frac{5 \cdot 4}{2} p^4 q^2 + \frac{6 \cdot 5 \cdot 4}{3 \cdot 2} p^4 q^3$$

$$\rightsquigarrow p^4 \left(1 + 4q + 10q^2 + 20q^3\right) \rightsquigarrow 0.39$$