W05 Homework A

Due date: Thursday 2/12, 11:59pm

01

Potholes on the highway

On a certain terrible stretch of highway, the appearance of potholes can be modeled by a Poisson process. Let the RV X denote the distance between successive potholes (measured in miles). The CDF of X is:

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ 1 - e^{-0.5x} & x > 0 \end{cases}$$

(a) What is the mean number of potholes in a 2-mile stretch of the highway?

(b) What is the probability that there will be at least 2 potholes in a 2-mile stretch of the highway?

02

Selling Christmas trees

An online company sells artificial Christmas trees. During the holiday season, the amount of time between sales, T, is an exponential random variable with an expected value of 2.5 hours.

(a) Find the probability that the store will sell more than 2 trees in a 1-hour period of time.

(b) Find the probability the time between the sales of two trees will be between 4-5 hours.

03

Constants in PDF from expectation

Suppose X has PDF given by:

$$f_X(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Suppose $E[X] = \frac{7}{10}$. Find the only possible values for a and b. Then find $\text{Var}[X]$.

04

Variance: Direct integral formula

Suppose X has PDF given by:

$$f_X(x) = \begin{cases} 3e^{-3x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $\text{Var}[X]$ directly using the integral formula.

05

Binomial - Repeated coin flips

A coin is flipped 7 times and the sequence of results recorded as an outcome.

(a) How many possible outcomes have exactly 3 heads?

(b) How many possible outcomes have at least 3 heads?