W05 Homework B

Due date: Tuesday 2/17, 11:59pm

01 ★

Application of Poisson: meteor shower

The UVA astronomy club is watching a meteor shower. Meteors appear at an average rate of 4 per hour.

(a) Write a short explanation to justify the use of a Poisson distribution to model the appearance of meteors. Why should appearances be Poisson distributed?

(b) What is the probability that the club sees more than 2 meteors in a single hour?

(c) Suppose we learn that over a four hour evening, 13 meteors were spotted. What is the probability that none of them happened in the first hour?

02 ★

Vehicle lifetimes

Suppose that vehicle lifetimes follow an exponential distribution with an expected lifetime of 10 years.

Suppose you have one car that is 5 years old, and one that is 15 years old, at the present moment.

What is the probability that the first car outlives the second? (I.e. that the second breaks at an earlier time than the first breaks, both starting now.)

03

Wait time for 5 calls - two methods

Consider the Poisson process of phone calls coming to a call center at an average rate of 1 call every 6 minutes.

Let us model the wait time for 5 calls to come in. You may use Desmos or similar to perform the integration numerically.

(a) Method One: An arrival of '1-call' comes in at an average rate of $\lambda = 10$ calls per hour. So a Bundle of '5-calls' comes in at an average rate of $\lambda_B = 2$ Bundles per hour. Use an exponential variable with $\lambda_B = 2$ to determine the probability that the wait time for a Bundle (of 5 calls) is at most $1\,\text{hr}$.

(b) Method Two: Use $\lambda = 10$ calls per hour with an Erlang distribution at $\ell = 5$ to determine the probability that the wait time for 5 calls is at most $1\,\text{hr}$.

(c) Compare the results of (a) and (b). Can you explain why they agree or disagree? Which is correct??

Mean and variance of exponential

Show that $E[X] = \frac{1}{\lambda}$ and $\mathrm{Var}[X] = \frac{1}{\lambda^2}$ for $X \sim \mathrm{Exp}(\lambda)$.

05 ★

Expectation from CDF

The CDF of random variable X is given by:

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+1}{2} & -1 \leq x < 0 \\ 1/2 & 0 \leq x < 2 \\ \dfrac{x-1}{2} & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

Compute $E[X]$.

06

PDF of derived variable for $E[X]$ and $\mathrm{Var}[X]$

Suppose the PDF of an RV is given by:

$$f_X(x) = \begin{cases} \frac{3}{4}x(2-x) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $E[X]$ using the integral formula.

(b) Find $f_{X^2}(x)$, the PDF of X^2 (by calculating the CDF first).

(c) Find $E[X^2]$ using $f_{X^2}(x)$.

(d) Find $\mathrm{Var}[X]$ using results of (a) and (c).

07

Rolling until a six

A fair die is rolled until a six comes up.

What are the odds that it takes at least 10 rolls? (Use a geometric random variable.)