W06 - Examples

Normal distribution

Basic generalized normal calculation

Suppose $X \sim \mathcal{N}(-3, 4)$. Find $P[X \geq -1.7]$.

Solution

First write X as a linear transformation of Z:

$$X \sim 2Z - 3$$

Then:

$$X \geq -1.7 \quad \iff \quad Z \geq 0.65$$

Look in a table to find that $\Phi(0.65) \approx 0.74$ and therefore:

$$P[Z \geq 0.65] \quad \gg\gg \quad 1 - P[Z \leq 0.65]$$

$$\gg\gg \quad \approx 1 - 0.74 \quad \gg\gg \quad \boxed{0.26}$$

Gaussian: interval of 2/3

Find the number a such that $P\big[-a \leq Z \leq +a\big] = 2/3$.

Solution

First convert the question:

$$\begin{aligned} P\big[-a \leq Z \leq +a\big] \quad &\gg\gg \quad F_Z(a) - F_Z(-a) \\ &\gg\gg \quad \Phi(a) - \Phi(-a) \\ &\gg\gg \quad 2\Phi(a) - 1 \end{aligned}$$

Solve for a so that this value is $2/3$:

$$2\Phi(a) - 1 = 2/3 \quad \gg\gg \quad \Phi(a) = 5/6 \quad \gg\gg \quad a = \Phi^{-1}(5/6)$$

Use a Φ table to conclude $a \approx 0.97$.

Heights of American males

Suppose that the height of an American male in inches follows the normal distribution $\mathcal{N}(71, 6.25)$.

(a) What percent of American males are over 6 feet, 2 inches tall?

(b) What percent of those over 6 feet tall are also over 6 feet, 5 inches tall?

Solution

(a)
Let H be a random variable measuring the height of American males in inches, so $H \sim \mathcal{N}(71, 2.5^2)$. Thus $H \sim 2.5Z + 71$, and:

$$\begin{aligned} P[H > 74] \quad &\ggg \quad 1 - P[H \leq 74] \\ &\ggg \quad 1 - P[2.5Z + 71 \leq 74] \\ &\ggg \quad 1 - P[Z \leq 1.20] \\ &\ggg \quad 1 - 0.8849 \approx 11.5\% \end{aligned}$$

(b)
We seek $P[H > 77 \mid H > 72]$ as the answer. Compute as follows:

$$\begin{aligned} P[H > 77 \mid H > 72] \quad &= \quad \frac{P[H > 77]}{P[H > 72]} \\ &\ggg \quad \frac{P[2.5Z + 71 > 77]}{P[2.5Z + 71 > 72]} \\ &\ggg \quad \frac{1 - P[Z \leq 2.4]}{1 - P[Z \leq 0.4]} = \frac{1 - 0.9918}{1 - 0.6554} \approx \boxed{2.38\%} \end{aligned}$$

Variance of normal from CDF table

Suppose $X \sim \mathcal{N}(5, \sigma^2)$, and suppose you know $P[X > 9] = 0.2$.

Find the approximate value of σ using a Φ table.

Solution

$$X \sim \mathcal{N}(5, \sigma^2) \implies X \sim \sigma Z + 5$$

So $1 - P[X \leq 9] = 0.2$ and thus $P[\sigma Z + 5 \leq 9] = 0.8$. Then:

$$P[\sigma Z + 5 \leq 9] \quad = \quad P[Z \leq 4/\sigma]$$

so $P[Z \leq 4/\sigma] = 0.8$.

Looking in the chart of Φ for the nearest inverse of 0.8, we obtain $4/\sigma = 0.842$, hence $\sigma = 4.75$.