W06 Homework

01

> ✎ **Gaussian basics**
>
> Find the probability that one observation of a Gaussian variable will yield a value within 1.5 standard deviations of the mean.

✍ Morning commute time

My morning commute time is normally distributed, with a mean of 14 minutes, and a standard deviation of 4 minutes. I leave for work every morning at 8:45am and need to arrive by 9:00am.

(a) On any given day, what is the probability that I am late?

(b) On any given day, what is the probability that I reach before 8:55am?

✍ Normal distribution - test scores

In a large probability theory exam, the scores are normally distributed with a mean of 75 and a standard deviation of 10.

(a) What is the probability that a student scored between 70 and 80

(b) What is the lowest score a student can achieve to be in the top 5%?

(c) What score corresponds to the 25th percentile?

☑ Generalized normal

Let X be generalized normal variable with $\mu = 3$ and $\sigma = 2$. Using a chart of Φ values, find:

(a) $P[2 < X < 6]$

(b) c such that $F_X(c) = 0.67$

(c) $E[X^2]$ (Hint: Use μ and σ to avoid integration.)

✎ Insurance expected payout

A car insurance analytics team estimates that the cost of repairs per accident is uniformly distributed between $100 and $1500. The manager wants to offer customers a policy that has a $500 deductible and covers all costs above the deductible.

How much is the expected payout per accident?

(Hint: Graph the PDF for the cost of repairs X; write a formula for the payout in terms of X using cases; then integrate.)

✎ CDF of derived variable

Suppose X is a continuous $\text{Unif}[1,4]$ random variable. Let $Y = |X - 2|$. Find the CDF of Y.

✎ Octane revenue

The owner of a small gas station has his 1,500 gallon tank of 93-octane gas filled up once at the beginning of each week. The random variable X is the amount of 93-octane the station sells in one week (in thousands of gallons). The PDF of X is shown below.

$$f_X(x) \;=\; \begin{cases} x & 0 \le x \le 1 \\ 1 & 1 < x \le 1.5 \end{cases}$$

Assuming the station consistently charges $3.00 per gallon for 93-octane and pays $2,000 for the weekly fill-up, find the CDF of $W = 3X - 2$, the profit the station makes from the 93-octane in a week.