

# W07 Homework B

Due date: Tuesday 3/10, 11:59pm

01 ★

## ✍ PMF calculations from a table

Suppose the joint PMF of  $X$  and  $Y$  has values given in this table:

$X \setminus Y$	0	1	2	3
1	0.10	0.15	0	0.05
2	0.20	0.05	0.05	0.20
3	0.05	0	$x$	0.05

- Find  $x$ .
- Find the marginal PMF of  $X$ .
- Find the PMF of the random variable  $Z = XY$ .
- Find  $P[X = Y]$  and  $P[X > Y]$ .

**✍ Marginals and probability from joint PDF**

Suppose  $X$  and  $Y$  have joint PDF given by:

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the marginal PDFs for  $X$  and  $Y$ .
- (b) Find  $P[X > Y]$ .

**✍ Air pollution**

In a certain community, levels of air pollution may exceed federal standards for ozone or for particulate matter on some days. In a particular summer week, let  $X$  be the number of days on which the ozone standard is exceeded, and let  $Y$  be the number of days on which the particulate matter is exceeded.

The following table represents the joint PMF for  $X$  and  $Y$ .

	$Y = 1$	$Y = 2$	$Y = 3$
$X = 1$	0.09	0.11	0.05
$X = 2$	0.17	0.23	0.08
$X = 3$	0.06	0.15	0.06

(a) Find  $P[X > Y]$ .

(b) Find  $E[X + Y]$ .

**✍ Composite PDF from joint PDF**

The joint density of random variables  $X$  and  $Y$  is given by:

$$f_{X,Y}(x,y) = \begin{cases} e^{-x-y} & x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Compute the PDF of  $X/Y$ .

**✍ One car outlasts the other**

Suppose that  $X, Y \sim \text{Exp}(0.1)$  are two independent exponential random variables.

(a) Find the joint PDF  $f_{X,Y}$ .

(b) Find the probability:

$$P[X > Y \mid X > 5 \text{ AND } Y > 15]$$

Compare to W05-B Q02 “Vehicle Lifetimes.” Which method is easier? (For yourself, not this HW.)

**✍ Normal distribution - cars passing toll booth**

The number of cars passing a toll booth on Wednesdays has a normal distribution  $\mathcal{N}(1200, 40000)$ .

- (a) What is the probability that on a randomly chosen Wednesday, more than 1,400 cars pass the toll booth?
- (b) What is the probability that between 1,000 and 1,400 cars pass the toll booth on a random Wednesday?
- (c) Suppose it is learned that at least 1200 cars passed the toll booth last Wednesday. What is the probability that at least 1300 cars passed the toll booth that day?