W09 Homework A

Due date: Thursday 3/12, 11:59pm

01

PDF of Min

Let X and Y be independent copies of a $\text{Unif}[0,1]$ random variable.

Let $W = \text{Min}(X, Y)$. Find the PDF of W.

02

PDF of Min and Max

Suppose $X \sim \text{Exp}(2)$ and $Y \sim \text{Exp}(3)$ and these variables are independent. Find:

(a) The PDF of $W = \text{Max}(X, Y)$

(b) The PDF of $W = \text{Min}(X, Y)$

03

PDF of sum of uniforms

Let X and Y be independent copies of a Unif$[0, 1]$ random variable. Let $W = X + Y$.

Find the PDF of W.

04

PDF of sum from joint PDF

Suppose the joint PDF of X and Y is given by:

$$f_{X,Y} = \begin{cases} \frac{8}{81}xy & 0 \leq y \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the PDF of $X + Y$.

05

Function on one variable

Suppose the PDF of X is given by:

$$f_X(x) = \begin{cases} \frac{2}{3}x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the CDF and PDF of $W = \ln X$.

06

Finish a PMF table - Strange families

Suppose that 15 percent of the families in a strange community have no children, 20 percent have 1 child, 35 percent have 2 children, and 30 percent have 3 children. Assume the odds of a child being a boy or a girl are equal.

If a family is chosen at random from this community, then B, the number of boys, and G, the number of girls, in this family will have the joint PMF partially shown in this table:

$B \downarrow \; G \rightarrow$	**0**	**1**	**2**	**3**	$P_B(i)$:
0	0.15	0.10	?	0.0375	0.3750
1	0.10	0.175	?	0.00	0.3875
2	?	?	0.00	0.00	0.2000
3	0.0375	0.00	0.00	0.00	0.0375
$P_G(j)$:	0.3750	0.3875	0.2000	0.0375	[not used]

(a) Complete the table by finding the missing entries.

(b) What is the probability that "B or G is 1"?

07

Soft-drink machine

A soft-drink machine has a random amount Y in supply at the beginning of a given day and dispenses a random amount X during the day (with measurements in gallons). It is not resupplied during the day, and therefore $X \leq Y$. It has been observed that X and Y have a joint density given by:

$$f_{X,Y}(x,y) \;=\; \frac{1}{2}, \qquad 0 \leq x \leq y \leq 2$$

(a) Find the probability that the amount of soft-drink dispensed on a given day is greater than 1.5 gallons. (First compute the marginal PDF of X.)

(b) Find the probability that the amount of soda remaining in the machine at the end of the day is greater than 1/2 gallon. That is, find $P[Y - X > 1/2]$.

(c) Find the CDF of $W = Y - X$, the amount of soda remaining in the machine at the end of the day.

Random point in a triangle

Consider a joint distribution that is uniform over the triangle with vertices $(0,0), (0,1)$, and $(1,0)$. Suppose a point (X,Y) is chosen at random according to this distribution.

(a) Find the joint PDF $f_{X,Y}$.

(b) Find the marginal PDFs for X and Y.

(c) Are X and Y independent?