

W09 Homework B

Due date: Tuesday 3/17, 11:59pm

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☑ Coffee and danish

Joe visits a coffee shop every morning and orders a triple espresso and a cherry danish. Let X represent the wait time for the espresso and Y the wait time for the danish (both in minutes). The joint PDF of X and Y is:

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{50} e^{-\frac{1}{5}x - \frac{1}{10}y} & x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that Joe will not sit down until he has both his espresso and his danish. Compute the probability that he will have to wait at least 5 minutes before sitting down, that is, find $P[\text{Max}(X, Y) > 5]$.

✍ Lights on

An electronic device is designed to switch house lights on and off at random times after it has been activated. Assume the device is designed in such a way that it will be switched on and off exactly once in a 1-hour period. Let X denote the time at which the lights are turned on and Y the time at which they are turned off. Assume the joint density for (X, Y) is given by:

$$f_{X,Y}(x, y) = 8xy, \quad 0 < x < y < 1$$

Let $W = Y - X$, the time the lights remain on during the hour.

- (a) Find the range of W .
- (b) Compute a formula for the CDF of W , i.e. $F_W(w)$.
- (c) Find the probability the lights remain on for at least 40 minutes in some given hour.

✍ Derived random variable

Let $U = \text{Unif}[0, 1]$ and suppose that $X = -\ln(1 - U)$.

- (a) Compute $F_X(x)$.
- (b) Compute $f_X(x)$.
- (c) Compute $E[X]$.

✍ Blood sugar testing

A test for diabetes is a measurement X of a person's blood sugar level following an overnight fast. If a person has diabetes, X is a Gaussian $(60, 40)$ random variable.

Now suppose that the doctor has decided to use the following scores: positive for diabetes if $X \geq 140$, negative if $X \leq 110$, and the test is inconclusive if $110 < X < 140$.

Find the probability the test result will be inconclusive for a person who has diabetes.

✍ Joint PMF with y -dependence

Suppose X and Y have the following joint PMF:

$$P_{X,Y}(x,y) = \begin{cases} \frac{.05}{y} (.95)^{y-1} & x = 1, 2, \dots, y; \quad y = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P[Y < 3]$.

(b) $P[X = 1] = .1577$. Find $P[X = 2]$.

✍ Sales rep making calls

A sales representative is responsible for selling a particular item. On a given day, he has time to make a sales pitch to up to 3 customers. His goal is to sell the item to 2 customers; if he is successful with the first two, he will not try to sell to the 3rd customer. The probability of any one customer purchasing the item is 0.6, independent of the others.

Let X be the number of customers to which he tries to sell the item and Y be the number of customers that purchase the item. Construct the joint PMF of X and Y .

✍ Alice and Bob meeting at a cafe

Alice and Bob plan to meet at a cafe to do homework together. Alice arrives at the cafe at a random time (uniform) between 12:00pm and 1:00pm; Bob independently arrives at a random time (uniform) between 12:00pm and 2:00pm on the same day. Let X be the time, past 12:00pm, that Alice arrives (in hours) and Y be the time, past 12:00pm, that Bob arrives (in hours). So $X = 0$ and $Y = 0$ represent 12:00pm.

(a) Consider $W = \text{Max}(X, Y)$, the time that will pass (beyond 12:00pm) before Alice and Bob will begin working together. Find the CDF of W .

(b) Find the probability Alice will have to wait more than 75 minutes for Bob.