

W11 Homework A

Due date: Friday 3/27, 11:59pm

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✍ Conditional density from joint density

Suppose that X and Y have joint probability density given by:

$$f_{X,Y}(x,y) = \begin{cases} \frac{12}{5}x(2-x-y) & x, y \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute $f_{X|Y}(x|y)$, for $y \in [0, 1]$.

(b) Compute $P[X > 1/2 | Y = y]$.

✍ Time till recharge

Let X denote the amount of time (in hours) that a battery on a solar calculator will operate properly before needing to be recharged by exposure to light. The function below is the PDF of X .

$$f(x) = \begin{cases} \frac{50}{6}x^{-3} & 2 < x < 10 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that a calculator has already been in use for 5 hours. Find the probability it will operate properly for at least another 2 hours.

✍ Conditional distribution and expectation from joint PMF

Suppose that X and Y have the following joint PMF:

$$P_{X,Y}(k, \ell) = \begin{cases} c & k = 1, 2, 3, 4; \quad \ell = 1, \dots, k \\ 0 & \text{otherwise} \end{cases}$$

Notice that the possibilities for ℓ depend on the choice of k .

First, show that $c = 1/10$. Then compute:

- (a) P_X (b) $P_{Y|X}$ (c) $E[Y | X = 4]$ (d) $E[Y | X]$