W11 Homework B

Due date: Tuesday 3/31, 11:59pm

01

From conditional to joint, and back again

Suppose we have the following data about random variables X and Y:

$$f_X(x) \;=\; \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{Y|X}(y|x) \;=\; \begin{cases} 2y/x^2 & 0 \le y \le x \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the joint distribution $f_{X,Y}(x,y)$.

(b) Find $f_{X|Y}(x|y)$.

02

Cashews in a can

A nut company markets cans of mixed nuts containing almonds, cashews, and peanuts. Suppose the net weight of each can is exactly 1 lb, but the weight contribution of each type of nut is random. Let X be the weight of almonds in a selected can and Y the weight of cashews. The joint PDF of X and Y is given below:

$$f_{X,Y}(x,y) \;=\; \begin{cases} 24xy & x, y \in [0,1],\; x+y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that the weight of cashews in a particular can is 0.5 lbs. Calculate the probability that the weight of almonds in this can is more than 0.3 lbs.

03

New and returning customers

A sales representative will randomly select and call 2 customers. The representative's goal is to get each customer to complete a satisfaction survey. Each of these customers is categorized as "new" or "returning." 70% of customers are new and 30% are returning. Let X be the number of new customers that are called.

(a) Construct the PMF of X, $P_X(x)$.

The probability of any new customer completing the survey is 0.15, and the probability of any returning customer completing the survey is 0.20. (Customers operate independently.) Let Y be the number of new customers that complete the survey.

(b) Construct $P_{Y|X}(y|0)$, $P_{Y|X}(y|1)$, and $P_{Y|X}(y|2)$. (You should construct 3 separate PMFs.)

(c) Construct the joint PMF of X and Y.

04

Conditional distribution and expectation from joint PDF

Suppose that X and Y have the following joint PDF:

$$f_{X,Y}(x,y) \;=\; \begin{cases} cxy & 0 < y < 1,\ 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$

Notice that the range of possibilities for x depends on the value of y.

First, show that $c = 8$. Then compute:

(a) f_X (b) $f_{Y|X}$ (c) $E[\,Y \mid X = 0.5\,]$ (d) $E[\,Y \mid X\,]$