

W13 Homework A

Due date: Friday 4/10, 11:59pm

01

✍ Deviation estimation - Exponential

Let $X \sim \text{Exp}(\lambda)$ with $\lambda = 0.5$.

- (a) Compute the Markov bound on $P[X > 5]$.
- (b) Compute the Chebyshev bound on $P[X > 5]$.
- (c) Find the exact value of $P[X > 5]$ and compare with yours answers in (a) and (b).

✍ Deviation estimation - How many samples required?

Suppose the expected value of a score on the Probability final exam is 80 and the variance is 10. Assume the students' scores are independent.

How many students must take the exam before the average score in the class is known to lie within 5 points of 80 with a probability of 90%? What about 95%?

✍ Deviation estimation - Factory production

Suppose a factory produces an average of 50 items per week.

- (a) How likely is it that more than 75 items are produced this week? (Find an upper bound.)
- (b) Suppose the variance is known to be 25. Now what can you say about (a)? (Hint: Monotonicity.)
- (c) What do you know about the probability that the number of items produced differs from the average by at most 10?

✍ Community college ages

At a community college, the mean age of the students is 22.3 years, and the standard deviation is 4 years. A random sample of 64 students is drawn.

- (a) Use Markov's Inequality to find an upper bound for the probability that the average age of the students in the random sample is more than 23 years.
- (b) What is the probability that the total age of the students in the random sample is less than 1472 years?

✍ Math contest scores

At a high school math competition, students take a test with 10 questions. Each question is worth one point and the probability of a student getting any one question correct is 0.55, independent of the other questions.

- (a) Find the variance of M_{15} , the average score for 15 students.
- (b) Use the Law of Large Numbers to find an upper bound for the probability that M_{15} is greater than 6.