

W13 Homework B

Due date: Tuesday 4/14, 11:59pm

01

Chebyshev

Suppose X is an RV with $E[X] = 17$ and $E[X^2] = 298$.

Use Chebyshev's inequality to find:

- (a) A lower bound for $P[10 < X < 24]$.
- (b) An upper bound for $P[|X - 17| \geq 16]$.

✍ Normal v. Chebyshev - Jewelry insurance

A jewelry insurance provider has 2500 customers. The expected payout to a customer each year is \$1000 with a standard deviation of \$900.

What premium should be charged to each customer to ensure that the premiums will cover the claims, with probability at least 99.9%?

- (a) Solve the problem using a normal approximation.
- (b) Solve the problem using Chebyshev's inequality.

✍ Train arrivals

The time between train arrivals at a station is exponentially distributed with a mean of 5 minutes. Therefore, the arrival time of the n^{th} train, T_n , can be represented as $X_1 + X_2 + \cdots + X_n$ where each X_i is independent of the others and is exponentially distributed with a mean of 5 minutes. Suppose there are enough customers waiting to fill 3 trains.

- (a) Find the mean and variance of T_3 , the time elapsed when the third train arrives.
- (b) Use Chebyshev's inequality to find an upper bound for the probability that more than 35 minutes will pass before all customers can board a train.

✍ Random walk forward

You play a game where you roll a die, and if the outcome is 1 or 2 you take a step forward, otherwise you take two steps forward. Let X_n be your position (measured in steps forward) after playing the game n times.

- (a) Estimate $P[X_{90} \geq 160]$ using a normal approximation for a certain relevant binomial distribution.
- (b) Find $\lim_{n \rightarrow \infty} P[X_n > 1.6n]$ and $\lim_{n \rightarrow \infty} P[X_n > 1.7n]$.

Hint: Rewrite the conditions into a form where you can apply the Law of Large Numbers.

✍ Body weights

Assume that body weights of men are Normally distributed with a mean of 170 pounds and a standard deviation of 30 pounds.

What is the body weight threshold separating the lightest 90% from the heaviest 10%?

✍ Normal approximation of the binomial - Double ones

Roll a pair of dice 10,000 times. Estimate the odds that the number of “snake eyes” (double ones) obtained is in the range $[290, 300]$.