W14 Homework A

Due date: Thursday 4/16, 11:59pm

01

Testing paperclips - Likelihood of error

A factory assembly line machine is cutting paperclips to length before folding. Each paperclip is supposed to be $3\,\text{in}$ long. The length of paperclips is approximately normally distributed with standard deviation $0.2\,\text{in}$.

(a) Design a significance test with $\alpha = 0.02$ that is based on the average of 5 measurements (sample mean). What is the rejection region? What is the probability of Type I error?

(b) What is the probability of Type II error, given that the average paperclip length on the machine is actually $3.1\,\text{in}$?

02

Testing a coin by flipping until heads

Design a significance test to test the hypothesis that a given coin is fair. You think it may be biased towards tails.

Your test runs the following experiment: flip the coin repeatedly until the first time a heads comes up. Let N be the flip number of the first heads. This N is your decision statistic.

Your test should have significance level $\alpha = 0.02$.

Which of these coins would pass your test?

- Two-headed coin
- Two-tailed coin
- Both
- Neither

03

Shipping time test

The number of days it takes for a package to arrive after being shipped with a particular company is a random variable, X. When the shipping process is operating at full capacity and delays are not common, the PMF of X is given in the following table:

x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$P_X(x)$	0.041	0.229	0.379	0.237	0.045	0.021	0.019	0.017	0.012

Design a significance test at the $\alpha = 0.03$ level that uses the value of X for one package to test the null hypothesis: the shipping process is operating at full capacity. You should clearly state which values of X are in the rejection region.

Identifying Uranium

You are testing gram samples of pure Uranium to see if they are enriched. You have a Geiger counter that counts a number of gamma rays that come from nearby fission events in 1 second intervals after you press the count button.

If the sample is enriched, you expect a Poisson distribution N of gamma rays in the counter with an average of 20. If the sample is not enriched (the null hypothesis), the average count will be 10.

(a) Design an ML test to decide whether it is ordinary (H_0) or enriched (H_1). What is A_0? What are the probabilities of Type I, Type II, and Total error?

(b) After running the test many times, you have noticed that 70% of the samples are ordinary, while 30% are enriched. Now design an MAP test. What is A_0? What are the probabilities of Type I, Type II, and Total error?

(c) Missing a bit of enriched Uranium is obviously a major problem. The damage to your reputation and pocketbook of missing enriched Uranium is 100$\times$ the damage caused by incorrectly labeling ordinary Uranium as enriched. Now design an MC test. What is A_0? What are the probabilities of Type I, Type II, and Total error?

(d) What is the expected cost of each application of the MC test, assuming the cost of a false alarm is $10,000? What is this number for the MAP test?

Medical testing

A doctor is planning to use a new, inexpensive medical test to detect a particular disease. The test score, X, tends to be higher for patients with the disease. The PMFs for the test score for patients with and without the disease are shown below. From a previously used, more expensive test, it is known that 20% of the population has this disease.

Patients without the disease:

x	**1**	**2**	**3**	**4**	**5**
$P_X(x)$	0.5	0.3	0.15	0.05	0

Patients with the disease:

x	**1**	**2**	**3**	**4**	**5**
$P_X(x)$	0.05	0.1	0.3	0.35	0.2

Design a binary hypothesis test that will minimize the doctor's probability of error. Let H_0: the patient does not have the disease and H_1: the patient does have the disease. Determine for which test scores the doctor should diagnose the patient as having the disease. Clearly denote which scores result in which decisions.