W14 Homework B

Due date: Tuesday 4/21, 11:59pm

01

Valves at various temperatures

The lifetime of a certain fuel injection valve is known to follow an exponential distribution, $X \sim \text{Exp}(\lambda)$, where $\lambda = \left(\frac{T}{300}\right)^2$ in failings per year and T is the ambient temperature in degrees Celsius.

Sometimes the valves fail a good deal more frequently than usual, possibly due to cracked gaskets used in construction. To detect failings from cracked gaskets, each day the following test is performed: ℓ valves are monitored in use at 100° C for the full $24\,\text{hr}$ day and the number N that fail is recorded.

(a) Suppose a significance test is designed such that it rejects the hypothesis "normal valves, no cracked gaskets" when just one (or more) fail the test. What is the significance level of this test, as a function of ℓ?

(b) How many valves would have to be tested at 150° C in order to achieve a significance of $\alpha = 0.2$? (Find ℓ using the function resulting from (a).)

(c) Is ℓ (to achieve $\alpha = 0.2$) increasing, decreasing, or constant with increasing test temperature?

02

Blue eyes

A redditor claims that 10% of people have blue eyes, but you think it is not that many. You work at the DMV for the summer, so you write down the eye color recorded on drivers' licenses of various people in the database.

(a) Suppose you record the eye color of 1000 people and let X be the number that are blue. If the rejection region is $\{X \leq 85\}$, what is the significance level of the test?

(b) Take again the experiment in (a). If you want a significance level of $\alpha = 0.01$, what should the rejection region be in your test?

(c) Suppose the fact is that 7% of people have blue eyes. How likely is it that your test in (b) rejects H_0?

03

Light bulbs

Light bulbs from box A (the null hypothesis) typically last $1000\,\text{hrs}$, and bulbs from box B last $500\,\text{hrs}$. You have some bulbs but don't know which box they came from. Bulb lifetimes are exponential.

It costs \$50 in processing if you mistakenly assign a B bulb to box A, and \$20 if you assign an A bulb to box B.

After working at this for a while, you observed that 60% of the bulbs you see come from box A, and the rest from box B.

Design a binary hypothesis test using MC design to make a decision rule to assign bulbs to boxes.

(a) What is A_0?

(b) What are the probabilities of Type I, Type II, and Total error?

(c) What is the expected cost for each application of the test?

04

Security screening

A metal detector for an event produces a reading, X, that varies between 0 and 10 according to the PDFs given below. (Note X is a continuous random variable.)

Based on the reading, a security guard will stop and search a person or let them pass. Suppose it is known that 10% of people passing through security are carrying metal objects.

$H_0 =$ a person is not carrying metal objects
$H_1 =$ a person is carrying metal objects

$$f_{X|H_0}(x) = \frac{10 - x}{50}, \quad 0 < x < 10$$

$$f_{X|H_1}(x) = \frac{x}{50}, \quad 0 < x < 10$$

Suppose it is 20 times worse to neglect searching someone who is carrying metal than to search someone who is not carrying metal. Design a minimum cost test that uses the value of the reading, X to decide whether the security guard will stop that person. Clearly state the decision rule.