

W15 Homework A

Due date: Thursday 4/23, 11:59pm

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✍ MSE of a derived variable

Suppose that $X \sim \text{Unif}[0, 2]$, and $Y = X^2$.

You are using the sample mean with 50 samples, namely $M_{50}(Y)$, to estimate $E[Y]$.

What is the mean square error of $M_{50}(Y)$? (Errors are deviations of this variable from $E[Y]$.)

✍ Estimates from joint PDF

Suppose X and Y have the following joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} \frac{6(y-x)}{27} & 0 \leq x \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find $f_X(x)$ and the blind estimate \hat{x}_B .
- (b) Compute \hat{x}_G , the MMSE estimate of X assuming the event $G = \{X < 3/2\}$.
- (c) Find $f_Y(y)$ and the blind estimate \hat{y}_B .
- (d) Compute \hat{y}_H , the MMSE estimate of Y assuming the event $H = \{Y > 3/2\}$.

✍ MMSE exact estimator from joint PDF

Suppose X and Y have the following joint PDF:

$$f_{X,Y}(x, y) = \begin{cases} 2(y+x) & 0 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is $\hat{x}_M(y)$, the MMSE estimate of X given $Y = y$?
- (b) What is $\hat{y}_M(x)$, the MMSE estimate of Y given $X = x$?

✍ MMSE linear estimator from joint PDF

Suppose X and Y have the following joint PDF:

$$f_{X,Y}(x, y) = \begin{cases} 2(y+x) & 0 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is $\hat{X}_L(Y)$, the MMSE linear estimator of X in terms of Y ?
- (b) What is $\hat{Y}_L(X)$, the MMSE linear estimator of Y in terms of X ?