W15 Homework B

Due date: Tuesday 4/28, 11:59pm

01

MMSE linear estimator from joint PMF

Suppose X and Y have the following joint PMF:

$Y \downarrow \ X \rightarrow$	-1	0	1
1	$\frac{1}{6}$	$\frac{1}{12}$	0
3	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$
5	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{8}$
7	0	$\frac{1}{12}$	$\frac{1}{6}$

(a) Find the minimal MSE linear estimator for X in terms of Y.

(b) What is the MMSE error for this linear estimator?

(c) Use (a) to estimate X given $Y = 1$ and $Y = 5$.

02

MMSE linear estimator from joint density

Consider this joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} \frac{3}{2}\left(x^2+y^2\right) & x,y \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the minimal MSE linear estimator for X in terms of Y?

(b) What is the linear estimate of X given $Y = 0.7$?

03

Telemetry signal

A telemetry signal, T, transmitted from a temperature sensor on a communications satellite is a Gaussian random variable with $E[T] = 0$ and $\text{Var}[T] = 9$. The receiver at mission control receives $R = T + X$, where X is a noise voltage independent of T with PDF:

$$f_X(x) = \begin{cases} 1/6 & -3 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

The receiver uses R to calculate a linear estimate of the telemetry voltage:

$$\hat{T}_L(R) = aR + b$$

(a) What is $E[R]$, the expected value of the received voltage?

(b) What is $\text{Var}[R]$, the variance of the received voltage?

(c) What is $\text{Cov}[T, R]$, the covariance of the transmitted voltage and the received voltage?

(d) What is the correlation coefficient $\rho_{T,R}$ of T and R?

(e) What are a^* and b^*, the optimum mean square values of a and b in the linear estimator?

(f) What is e_L^*, the minimum mean square error of the linear estimate?

04

Bits received in error

In a digital communication channel, it is assumed that a bit is received in error with probability 4.3×10^{-5}. Someone challenges this hypothesis: they believe the error rate is higher than 4.3×10^{-5}. Assume 100,000 bits are transmitted. Design a one-tailed significance test using $\alpha = 0.05$ and N, the number of bits received in error, to decide whether to reject the hypothesis that the error rate is 4.3×10^{-5}. Your rejection region should be of the form $\{N \geq c\}$. You do not have to use the continuity correction.

05

CAT scan for tumors

When a brain is scanned in a CAT scan, analysis of the results yields a rating of 1, 2, 3, or 4. This represents (imperfect) evidence of whether there is a tumor.

	X	**1**	**2**	**3**	**4**
No tumor:	$P_X(k)$	0.4	0.3	0.2	0.1
	X	1	2	3	4
With tumor:	$P_X(k)$	0.0	0.1	0.3	0.6

Suppose that, of people who get CAT scans, 20% do have a tumor.

Furthermore, assume that declaring there is no tumor when there is one is ten times worse than declaring there is a tumor when there isn't one.

Design an MC test to determine which ratings should be classified as tumors.